LEEDS PHILOSOPHICAL AND LITERARY SOCIETY.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LVII. MARCH 12, 1897. No. 5

SIR R. S. BALL, LL.D., F.R.S., PRESIDENT, in the Chair.

J. Harold Herbert, C.E., Hazlemont, The Boulevard, Nottingham, was balloted for and duly elected a Fellow of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Arthur W. Clayden, M.A., F.G.S., F.C.S., F.R.Met.Soc., Principal of the Exeter Technical and University Extension College (proposed by Wm. Ellis) ;

Herbert Lovis Noel Noel-Cox, 2 Edinburgh Mansions, Victoria Street, S.W. (proposed by Richard Inwards).

One hundred and thirty-seven presents were announced as having been received since the last meeting, including amongst others :—

G. C. Comstock, studies in spherical and practical astronomy ; W. T. Lynn, Celestial Motions, 9th edition ; Remarkable Comets, 5th edition ; Remarkable Eclipses, 2nd edition ; T. J. J. See, Researches on the evolution of the stellar systems, vol. i. ; presented by the authors ; Report of the observations of the total solar eclipse 1896 August 9, in Novaya Zemlya, and lantern slides of the Corona, from negatives by MM. Kostinsky and Hansky, presented by M. Kostinsky ; Series of 12 enlarge-

ments from negatives of portions of the lunar surface, presented by the Lick Observatory ; Photographs of the Great Sun-spot of 1897 January, presented by G. J. Newbegin.

Also twenty-five works from the library of the late Mr. Sidney Waters, and an equatorially-mounted camera with telephotographic lens, prepared for the observation of the total eclipse of 1896, presented by Mrs. Waters.

On the Mean Motions of the Lunar Perigee and Node.
By Professor Ernest W. Brown, M.A.

The following is a brief abstract of the paper :

The calculations necessary for obtaining a new development of the lunar theory having progressed to a stage at which it is possible to obtain very approximate values of the mean motions of the perigee and node of the Moon, it is of some interest to see how the results compare with those of Delaunay and Hansen. The motions are now obtained so as to include the second powers of the eccentricities and inclination. The remaining parts are estimated from Delaunay's results. Using Dr. Hill's values for the parts due to the figure of the Earth, the results are :

Mean annual motion of Perigee	$+146\ 435''\cdot3$,
„ „ „ Node	$-\ 69\ 679''\cdot5$;

with a possible error not exceeding $2''$ in the case of the perigee, and $1''$ in the case of the node. Hansen's results, when corrected for the part depending on the figure of the Earth, give

$$+146\ 434''\cdot9,$$
$$-\ 69\ 677''\cdot3.$$

The observations give

$$+146\ 435''\cdot6,$$
$$-\ 69\ 679''\cdot5.$$

It appears that the difference of $2''\cdot2$ which Hansen found in the case of the node is at least partly due to some error. Nothing can be said as yet concerning the question as to whether the Newtonian law is exact, an hypothesis treated by Professor Newcomb in his "Astronomical Constants." He finds that a difference of $+1''\cdot40$ would result in the case of the perigee. The corrected value of Hansen gives only $+0''\cdot7$, while that which I have obtained is $+0''\cdot3$, but the error of the last result may possibly be as great as $2''$. The question awaits the deter-

mination of the higher terms due to the Sun's action—a determination which I hope to make in the course of a year or two.

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1. Up to the present time, the only theoretical values of the mean motions of the lunar perigee and node which even pretend to be accurate to one seventy-thousandth part of the whole, are those derived by Hansen. Delaunay's algebraical developments, though carried in some cases to a high power of the ratio of the mean motions, are quite insufficient, and doubts have been thrown on the accuracy of his higher terms.

Apart from the question of the formation of tables for the Moon's motion, there is the interesting one which Professor Newcomb has brought forward in his "Astronomical Constants" as to whether the Newtonian law of gravitation requires any modification. The supposition made is that the force with which one body attracts another does not vary as the exact inverse second power of the distance, but that the index differs from -2 by some very small fraction. By assuming the index to be $-2{\cdot}0000001574$, the differences between theory and observation in the cases of the perihelia of *Mars* and *Mercury* are fairly well accounted for (*op. cit.* p. 119), the effects on those of the other planets not being at present within the range of observation. But the same number will cause an addition to the annual mean motion of the lunar perigee of $+1''{\cdot}40$; the effect on the motion of the nodes is insensible. The comparison of the numbers given by Hansen shows a difference of $+1''{\cdot}56$ for the perigee, but Professor Newcomb adds that the argument loses all sustaining force on account of a difference of nearly $3''$ in the case of the node. Thus, either Hansen's values are not accurate, or some other cause affects the motion of the node.

2. My calculations of the solar inequalities in the Moon's motion having reached a stage at which it is possible to obtain the theoretical values of these mean motions accurately as far as the second order inclusive, in relation to the eccentricities and inclinations, it is interesting to see, first, how Delaunay's algebraical results appear as to accuracy and convergence; secondly, how Hansen's numerical results appear as to accuracy; thirdly, whether any fresh light can be thrown on the hypothesis mentioned above. In the new results given below, the question of convergence, from a practical point of view, does not enter at all; for the results are obtained by using the *numerical* value of the ratio of the mean motions of the Sun and Moon (a ratio known from observation with great accuracy), the values of the other constants only being inserted at the end of the work. The series for the quantities under consideration here when arranged according to powers of the squares of the eccentricities, inclination, and ratio of the parallaxes, diminish so rapidly that it is quite unnecessary to proceed further than the fourth powers of these constants, while those of this order are very small.

3. In order that the degree of accuracy obtained may be clearly defined, I give below a detailed comparison of the new results with those obtained by Delaunay, and also a comparison of the whole with those derived by Hansen from theory and observation. Delaunay's numerical results being given in terms of the *daily* motions, I use the latter for the discussion of them, returning to *annual* motions when the final general values are to be considered. The word "order" used below refers only to powers of the eccentricities, inclination, and the square root of the ratio of the parallaxes (this ratio being really a small quantity of the second order), and not to powers of the ratio of the mean motions.

4. As the terms of the fourth order are as yet undetermined (using the word "order" in the sense just explained), it is necessary to deduce their values from the results given by Delaunay. That these may be as accurate as possible, before proceeding to the main discussion, I give a test of their accuracy. Estimation is made of the remainders in each case. It is right to state here that these estimations were made in such a way that any conscious or unconscious influence which a knowledge of the result required for agreement with observation would have caused, was eliminated by one or two artifices. The possible errors have intentionally been made large, so that no false idea as to the accuracy of the final results might be entertained.

5. Although no positive evidence for or against the hypothesis treated by Professor Newcomb is obtained, one result seems to be demonstrated, namely, that the value obtained by Hansen for the annual motion of the node is at least one, if not two, seconds in error. Further, if his value for the part of the motion of the perigee which depends on the solar action is correct, one half of the small difference between theory and observation can be accounted for by using Dr. Hill's instead of Hansen's value for the part depending on the figure of the Earth. This, however, tells nothing definite, for if doubts are thrown on the accuracy of Hansen's calculations for the motion of the node, we are not entitled to assume the accuracy of the result for the perigee. The most probable theoretical values for both these motions are given at the end of the paper.

A Test of Delaunay's Expressions.

6. In a paper which will shortly appear in the Proceedings of the London Mathematical Society, the following results are shown to hold when the square of the ratio of the solar to the lunar parallax is neglected.

$$c_1 + c_2\frac{\partial \pi_1}{\partial n} + c_3\frac{\partial \theta}{\partial n} = \frac{3}{2}(\mathrm{E}+\mathrm{M})\frac{\partial}{\partial n}\left(\frac{1}{r}\right)_0,$$

$$c_2\frac{\partial \pi_1}{\partial \cdot e^2} + c_3\frac{\partial \theta_1}{\partial \cdot e^2} = \frac{3}{2}(\mathrm{E}+\mathrm{M})\frac{\partial}{\partial \cdot e^2}\left(\frac{1}{r}\right)_0,$$

$$c_2\frac{\partial \pi_1}{\partial \cdot \gamma^2} + c_3\frac{\partial \theta_1}{\partial \cdot \gamma^2} = \frac{3}{2}(\mathrm{E}+\mathrm{M})\frac{\partial}{\partial \cdot \gamma^2}\left(\frac{1}{r}\right)_0.$$

Here c_1, c_1+c_2, $c_1+c_2+c_3$ are the values of Delaunay's final variables L, G, H, after all the periodic terms have been eliminated ;* π_1, θ_1 are the mean motions of the perigee and node ; $E+M=n^2a^3$ is the sum of the masses of the Earth and Moon, and $\left(\frac{1}{r}\right)_0$ denotes the constant term in the value of $1/r$ when the latter is exhibited as a sum of periodic terms depending on the time. All the quantities are supposed to be expressed in terms of Delaunay's final constants n, a, e, γ, e', a', n'.

With the help of the expression for the constant part of the parallax given by Adams, † these equations may be used to verify Delaunay's results. The values of L, G, H, and thence those of c_1, c_2, c_3 in terms of the final constants, have been deduced by Professor Newcomb.‡ Since Delaunay does not make use of any similar verification equations, I have thought it worth while to test his results as far as it was possible with Adams' expression. The highest power of m tested is given in each case.

Test of the values of c_1, c_2, c_3, given by Professor Newcomb on p. 184 of the memoir just referred to. The value of c_1 is tested completely except for the part depending on a^2/a'^2. The value of c_2 contains e^2 as a factor of all its terms. It is tested as far as e^2m^3, e^4m^1 ; there are no terms involving $e^2\gamma^2m^0$, $e^2\gamma^2m^1$, $e^2e'^2m^0$, $e^2e'^2m^1$. The value of c_3 contains γ^2 as a factor of all its terms. It is tested as far as γ^2m^3, $\gamma^2e^2m^1$; there are no terms involving $\gamma^2e'^2m^0$, $\gamma^2e'^2m^1$, γ^4m^0, γ^4m^1.

We now come to the motions of the perigee and node. It is only possible to test these to a low power of m in any case. Every term tested was found correct. As the numerical values of the terms which contain e^2, γ^2, e'^2 as factors are given below, no detailed statement is necessary. But as Delaunay's values for the terms of higher orders are used, I give the results obtained.

The equation most convenient for these terms is

$$\frac{\partial c_2}{\partial \cdot \gamma^2}\cdot\frac{\partial \pi_1}{\partial \cdot e^2}+\frac{\partial c_3}{\partial \cdot \gamma^2}\cdot\frac{\partial \theta_1}{\partial \cdot e^2}-\frac{\partial c_2}{\partial \cdot e^2}\cdot\frac{\partial \pi_1}{\partial \cdot \gamma^2}-\frac{\partial c_3}{\partial \cdot e^2}\cdot\frac{\partial \theta_1}{\partial \cdot \gamma^2}=0,$$

which is deducible directly from the second and third of the equations given above.

In the motion of the perigee the terms involving $e^2\gamma^2m^2$, $e^2\gamma^2m^3$, γ^4m^2, γ^4m^3, $\gamma^2e^2m^2$, $\gamma^2e^2m^3$, $\gamma^2e'^2m^2$, $\gamma^2e'^2m^3$, were tested and found correct. In the motion of the node, the terms involving e^4m^2, e^4m^3, $e^2\gamma^2m^2$, $e^2\gamma^2m^3$, $e^2e'^2m^2$, $e^2e'^2m^3$ were tested and found correct. It will be noticed that these terms include all the larger terms given in the tables below for terms of the fourth order, except those depending on a^2/a'^2.

* *Théorie de la Lune*, vol. ii. pp. 235, 236.

† *M.N.R.A.S.* vol. xxxviii. *Coll. Works*, p. 203.

‡ Papers published for the use of the Amer. Eph. vol. v. pp. 184, 201, 202.

The Motion of the Perigee.

7. *Term of Zero Order.*—This has been completely obtained and discussed by Dr. G. W. Hill * and M. Andoyer. † The value which results from the numerical determination made by the former is,

Daily Motion, $+406''\cdot6402$.

8. *Terms of the Second Order.*—The following table gives the daily motions obtained by Delaunay, ‡ arranged according to the powers of m, the ratio of the mean motions. As usual, e, γ, e' are the constants defined in Delaunay's manner.

Daily Motions.

Factors.	e^2	γ^2	e'^2
	$''$	$''$	$''$
m^2	$-0\cdot3000$	$-3\cdot2085$	$+0\cdot0840$
m^3	$-\cdot6311$	$-\cdot9450$	$+\cdot1440$
m^4	$-\cdot2763$	$-\cdot3705$	$+\cdot0998$
m^5	$-\cdot1213$	$-\cdot1466$	$+\cdot0539$
m^6	$-\cdot0387$	...	...
m^7	$+\cdot0006$	...	...
Sum	$-1\cdot3668$	$-4\cdot6706$	$+0\cdot3817$
Est. Rem.	$+\cdot004$	$-\cdot07$	$+\cdot05$
Delaunay	$-1\cdot36$	$-4\cdot74$	$+0\cdot43$
True value	$-1\cdot4218$	$-4\cdot7627$	$+0\cdot4278$

The part contributed by the term of any order in relation to e^2, γ^2, e'^2, m^i is seen in this table. Thus the term of order $m^4\gamma^2$, obtained by Delaunay, furnishes $-0''\cdot3705$. The line "Est. Rem." is the estimated sum of the remaining terms of Delaunay's expressions. The next line gives the resulting value which is thus obtained, and in the last line are the correct values resulting from my calculations. When the possible errors of estimation are taken into account, the agreement is fairly satisfactory, except in the case of the terms depending upon e^2.

The results of the calculations for these latter are given in my paper 'On the Elliptic Inequalities in the Lunar Theory.' § Feeling that the discordance was too great to rest on a single determination, I recalculated the whole of the inequalities depending on e^2 by my later method. ‖ An almost exact agreement with the earlier result was found, showing that certainly Delaunay's coefficient for e^2m^7, and perhaps that for e^2m^6, are

* *Acta. Math.* vol. viii. pp. 1–36. *Annals of Math.* vol. ix. pp. 31–41.

† *Ann. de la Fac. des Sc. de Toulouse*, vol. vi.

‡ *C. R.* vol. lxxiv. pp. 19 *et sq.*, where these and the following results will be found.

§ *Amer. Jour. Math.* vol. xv. pp. 244–263, 321–338.

‖ *Ibid.* vol. xvii. pp. 318–358.

seriously wrong. It will be noticed that the small estimated remainder has the positive sign, whereas the true value shows that it should be a comparatively large quantity with the negative sign.

9. *Terms of the Fourth Order.*—The values of these must be obtained from Delaunay's expressions. The manner in which his method is carried out, taken in conjunction with the tests considered in § 6, give us a reasonable assurance of their accuracy; doubt only arises from the estimated remainders.

Daily Motions.

Factors.	e^4	γ^4	e'^4	a^2/a'^2
	"	"	"	"
m^2	−0·0002	−0·0121	+0·0000	+0·0024
m^3	...	+0·0056	...	+0·0019
Est. Rem.	−0·0004	+0·0040	+0·0001	+0·0020
Delaunay	−0·0006	−0·0025	+0·0001	+0·0063

Daily Motions.

Factors.	γ^2e^2	$\gamma^2e'^2$	$e^2e'^2$
	"	"	"
m^2	+0·0139	−0·0013	−0·0001
m^3	+0·0003	−0·0010	−0·0006
Est. Rem.	−0·0020	−0·0010	−0·0010
Delaunay	+0·0122	−0·0033	−0·0017

The sum of these gives

Daily Motion, +0″·0105,

with a possible error* due to wrong estimation of ± 0″·005. It will be noticed that the error arises chiefly from the terms which contain γ^4, γ^2e^2, a^2/a'^2 as factors.

Annual Motion of the Perigee.

10. Gathering together the true values given in §§ 7, 8, and the result obtained in § 9, we multiply them by 365·25 to reduce them to annual motions in a Julian year. The portions due to the figure of the Earth and to planetary action must also be included. The following are the results, with the authorities for them.

* No technical sense is to be attached to this expression.

Annual Motions.

Terms of the form $nf(m)$	$+148\ 525''3$	(Hill)
„ „ „ $ne^2f(m)$	$-\ 519''3$	(Brown)
„ „ „ $n\gamma^2f(m)$	$-\ 1\ 739''6$	„
„ „ „ $ne'^2f(m)$	$+\ 156''2$	„
„ „ higher orders	$+\ 3''8 \pm 1''8$	(Delaunay)
From the solar action	$+146\ 426''4 \pm 1''8$	
Figure of the Earth	$+\ 6''8$	(Hill)*
Planetary action	$+\ 2''1$	(Hansen)†
From theory	$+146\ 435''3 \pm 1''8$	. . . . . (i)
From observation	$+146\ 435''6$	(Hansen)† . . (ii)
From Hansen's theory	$+146\ 434''0$	„ . . . (iii)

Hansen, however, has obtained for the inequalities depending on the figure of the Earth the value $5''9$. Accepting the value obtained in the very extended and accurate investigation made by Dr. Hill, we must add $+\ 0''9$ to Hansen's result, making it

$$+146\ 434''9. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(iv)}$$

The evidence of agreement with observation afforded by the comparison of the values (i), (ii), is unfortunately rendered useless by the large possible error of $\pm\ 1''8$, which I have felt compelled to assign to the estimations of the higher terms due to the solar action. Hansen's determination of the part due to planetary action is probably correct within two or three tenths of a second.

If Hansen's result for the solar part be correct and we accept the value (iv), which at this stage is the best theoretical determination, it would seem that the hypothesis considered by Professor Newcomb is hardly verified in the case of the Moon. The required difference is $1''4$, while that furnished by the values (ii) and (iv) above is only $0''7$. The question must remain unanswered until an exact determination of the solar inequalities has been made. Such a determination is now in progress, and I hope may be finished before the lapse of any very considerable time, seeing that the outstanding parts, which are the chief cause of the possible error, are those easiest to calculate.

Motion of the Node.

11. *Term of Zero Order.*—This has been calculated to a high degree of accuracy by Professor J. C. Adams ‡ and Mr. P. H.

* Papers published for the Amer. Eph. vol. iii. p. 344.

† Hansen's results given here and in § 14, are found in Arts. 103, 159, 174 of the *Darlegung*.

‡ *M.N.R.A.S.* vol. xxxviii. pp. 43–49.

Cowell.* The agreement with Delaunay's result is quite satisfactory. From the value of the latter there results

Daily Motion, $-189''\cdot7005$.

12. *Terms of the Second Order.*—The results are exhibited in the same way as in § 8. The portion depending on γ^2 is extracted from Mr. Cowell's paper just referred to ; those depending on e^2, e'^2, are the results of my calculations.

Daily Motions.

Factors.	e^2	γ^2	e'^2
m^2	$-1''\cdot1999$	$+0''\cdot8021$	$-0''\cdot0840$
m^3	$-0''\cdot3534$	$-0''\cdot0675$	$+0''\cdot0040$
m^4	$-0''\cdot0958$	$-0''\cdot0197$	$+0''\cdot0053$
m^5	$-0''\cdot0270$	$-0''\cdot0016$	$+0''\cdot0022$
Sum	$-1''\cdot6761$	$+0''\cdot7133$	$-0''\cdot0725$
Est. Rem.	$-0''\cdot010$	$+0''\cdot000$	$+0''\cdot0009$
Delaunay	$-1''\cdot686$	$+0''\cdot713$	$-0''\cdot072$
True value	$-1''\cdot6865$	$+0''\cdot7134$	$-0''\cdot0697$

The agreement in all cases is sufficiently close. †

13. *Terms of the Fourth Order.*—As in the case of the perigee, we obtain these from Delaunay's expressions.

Daily Motions.

Factors.	e^4	γ^4	e'^4	a^2/a'^2
m^2	$+0''\cdot0008$	[none]	$-0''\cdot0000$	$-0''\cdot0024$
m^3	$-0''\cdot0005$	$+0''\cdot0001$	...	$-0''\cdot0005$
Est. Rem.	$-0''\cdot0003$	...	...	$-0''\cdot0001$
Delaunay	$+0''\cdot0000$	$+0''\cdot0001$	$-0''\cdot0000$	$-0''\cdot0030$

Daily Motions.

Factors.	$\gamma^2 e^2$	$\gamma^2 e'^2$	$e^2 e'^2$
m^2	$-0''\cdot0103$	$+0''\cdot0003$	$-0''\cdot0005$
m^3	$+0''\cdot0043$	$-0''\cdot0001$	$-0''\cdot0004$
Est. Rem.	$+0''\cdot0030$	$-0''\cdot0002$	$-0''\cdot0004$
Delaunay	$-0''\cdot0030$	$+0''\cdot0000$	$-0''\cdot0013$

The sum of these gives

Daily Motion $-0''\cdot007$,

with a possible error due to wrong estimation of $\pm''\cdot003$. Nearly the whole of this latter quantity arises in the estimation of the terms depending on $\gamma^2 e^2$.

* *Amer. Jour. Math.* vol. xviii. pp. 99–127.

† See the Postscript to this paper.

Annual Motion of the Node.

14. Proceeding as before, we find the following :—

Terms of the form $nf(m)$	$-69\ 288''\cdot1$	(Adams & Cowell)
„ „ „ $ne^2f(m)$	$-\ 616\cdot0$	(Brown)
„ „ „ $n\gamma^2f(m)$	$+\ 260\cdot6$	(Cowell)
„ „ „ $ne'^2f(m)$	$-\ 25\cdot5$	(Brown)
„ „ higher orders	$-\ 2\cdot6 \pm 1''\cdot1$	(Delaunay)
From solar action	$-69\ 671\cdot6 \pm 1''\cdot1$	
From the figure of the Earth	$-\ 6\cdot4$	(Hill)
From planetary action	$-\ 1\cdot5$	(Hansen)
From theory †	$-69\ 679\cdot5 \pm 1''\cdot1$	. . . . . (i)
From observation	$-69\ 679\cdot5$	(Newcomb)* . . . (ii)
From Hansen's theory	$-69\ 676\cdot8$	. . . . . (iii)

Hansen obtains for the inequality arising from the figure of the Earth the value $-5''\cdot9$. Accepting Dr. Hill's value, we find for Hansen's theoretical value

$$-69\ 677\cdot3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (iv)$$

The close agreement between the results (i) which I have deduced from theory and (ii) deduced from observation must be considered in connection with the possible error $\pm 1''\cdot1$. The corrected result of Hansen (iv) still differs from that given by observation by $2''\cdot2$. It appears, then, that the new theoretical value is rather more accurate than that of Hansen. It is probable that when the remainder of the solar inequalities in the motion of the node have been determined, an almost perfect agreement will be found between theory and observation.

15. While this determination is in progress, it would seem from the above discussion that the most accurate theoretical values as yet obtained are the result (iv) in the case of the perigee and the result (i) in the case of the node; that is to say,

Annual Mean Motion of the Perigee $+146\ 434''\cdot9$,
Annual Mean Motion of the Node $-\ 69\ 679''\cdot5$,

the amount by which these values may differ from the correct

* This value was kindly communicated to me by Professor Newcomb. It differs from Hansen's value by $0''\cdot1$.

† This result was announced at the December meeting of the American Mathematical Society.

theoretical values being not more than $\frac{1}{70,000}$ part of the whole in each case.

Haverford College, Pa.:
1897 *January* 5.

Postscript.—In the course of obtaining a verification of Delaunay's expressions for the secular accelerations, an error in the *Théorie* was discovered in the following way. The value of the part of the motion of the node which depends on e'^2, as given by Delaunay, is

$$-ne'^2\left(\frac{9}{8}m^2-\frac{23}{32}m^3-\frac{3261}{256}m^4-\frac{73423}{1024}m^5\right).$$

This value failed to furnish correctly the terms in the secular acceleration of the node which have the factors m^3, m^5, and, in that of the mean motion, the factors $m^3\gamma^2$, $m^5\gamma^2$. At first the natural assumption was that these four terms were wrong; but, on reconsidering Delaunay's method of obtaining them, this did not seem probable. I found that *all* the terms could be made to agree if we had

$$-ne'^2\left(\frac{9}{8}m^2-\frac{33}{32}m^3-\frac{3261}{256}m^4-\frac{73413}{1024}m^5\right)$$

instead of the above expression.

On referring to de Pontécoulant, *Système du Monde*, vol. iv. p. 567, I found that he gave 33 instead of 23 as the numerator of the second term. Moreover, the small difference between my result and that of Delaunay, as shown in the last column of the table in § 12 above, is exactly accounted for when the change is made.

It then seemed worth while to see if an error in Delaunay's *Théorie* could be detected without much trouble, and it was almost immediately found to be a simple error of transcription. The term

$$m'\frac{a^2}{a'^3}\left\{\ldots\left(\ldots+\frac{33}{16}\gamma^2e'^2\ldots\right)\frac{n'}{n}\ldots\right\},$$

in the value of the constant part of R on p. 2 of vol. ii. is incorrectly transcribed

$$m'\frac{a^2}{a'^3}\left\{\ldots\left(\ldots+\frac{23}{16}\gamma^2e'^2\ldots\right)\frac{n'}{n}\ldots\right\},$$

in the more extended value on p. 234, and the error is retained in the values of $\frac{dl}{dt}$, $\frac{dg}{dt}$, $\frac{dh}{dt}$, which immediately follow. This accounts for the error in the second term of the expression given above, and doubtless for that in the fourth term also.

On the Theoretical Values of the Secular Accelerations in the Lunar Theory. By Ernest W. Brown.

(*Abstract.*)

The accurate determination of certain inequalities in the Moon's motion has made it possible to find the theoretical values of the secular accelerations with a high degree of accuracy. The method of determination consists in using the theorem of Professor Newcomb for finding the secular accelerations in combination with that of the author connecting the mean motions of the perigee and node with the constant part of the lunar parallax. The resulting values, with the greatest possible amounts by which they are uncertain (assuming Newcomb's value for $\delta e'$ to be correct), are :

Secular accelerations per century of the

Mean motion	+ 5″·91	±0″·02,
Perigee	−38·9	±0·1,
Node	+ 6·56	±0·02.

1. The theoretical determination of the secular accelerations, formerly one of the most difficult and tedious problems in the lunar theory, has been rendered quite simple by the researches of Professor Newcomb.* Until the publication of these, two years ago, the only investigation with any approach to high accuracy was that of Delaunay, who carried the approximations so far as to include a great number of terms. The enormous labour of finding these, however, was hardly compensated by the result obtained, owing to the uncertainty produced by the slow convergence of the series when arranged according to powers of m. Hansen's results are entirely numerical, and he used those furnished by observation in the construction of his tables. In Newcomb's memoir, just referred to, the values are deduced from Delaunay's algebraical expressions as obtained in the *Théorie de la Lune*, but these again suffer from slow convergence, and errors must occur, first, from incorrect estimations of the remainders, and, secondly, from errors in Delaunay's theory. Owing to the former cause, Professor Newcomb concludes that his value (5″·60) for the secular acceleration of the mean motion is uncertain by more than $\frac{1}{3}\frac{1}{0}$ of the whole. He adds (mem. cit. p. 199) : "Although all the observations hitherto made on the Moon would not enable us to detect errors of this magnitude, a more accurate determination would be desirable from a theoretical point of view." It is the

* Papers published for the use of the *Amer. Eph.* vol. v. pt. iii.

object of this paper to show how such a determination may be made, and to find the values correctly to $\frac{1}{300}$ part of the whole in each case.

This has been rendered possible in three ways: First, by Newcomb's theorem, showing how Delaunay's final constants L, G, H may be calculated from the expressions for the rectangular coordinates of the Moon; secondly, by my theorem connecting the constant term in the expression for the parallax with L, G, H and the motions of the perigee and node;* and, thirdly, by the new determinations of these mean motions, the results of which have been given in my paper "On the Mean Motions of the Lunar Perigee and Node."† These theorems have also been employed to test Delaunay's algebraical expressions for the secular accelerations, and the results are given in § 4 below. In order to find the numerical values, such expressions as $\frac{\partial^2 \pi}{\partial n \partial . e'^2}$ have to be calculated; it is shown in § 5 how they may be found with very fair accuracy when the complete numerical value of $\frac{\partial \pi}{\partial e'^2}$ and only a few terms of its slowly convergent algebraical expression are known. Finally, the numerical results are given, with the amounts by which they are uncertain.

General Formulæ for the Secular Accelerations.

2. Professor Newcomb's theorem (mem. cit. p. 191) states that the variations of n (or a), e, γ due to a variation $\delta e'$ of e' may be obtained accurately to the first order, with respect to δ, by solving the equations

$$\delta L = 0, \quad \delta G = 0, \quad \delta H = 0,$$

or

$$\frac{\partial L}{\partial n}\delta n + \frac{\partial L}{\partial . e^2}\delta . e^2 + \frac{\partial L}{\partial . \gamma^2}\delta . \gamma^2 = -\frac{\partial L}{\partial . e'^2}\delta . e'^2, \text{ \&c.};$$

and that the secular accelerations of the mean motion, the perigee and node, are then given by

$$\delta\epsilon = \int \delta n \, dt, \quad \delta\pi = \int \delta\pi_1 \, dt, \quad \delta\theta = \int \delta\theta_1 \, dt,$$

where π_1, θ_1 are the mean motions of the perigee and node. Putting $L = c_1$, $G - L = c_2$, $H - G = c_3$, the first three equations become

$$\delta c_1 = 0, \quad \delta c_2 = 0, \quad \delta c_3 = 0.$$

* *Proc. London Math. Soc.* 1897.
† *Monthly Notices R.A.S.* 1897.

I have shown in the latter of the two papers just referred to that the variations of n, e, γ, e' also satisfy the equations

$$c_2\,\delta\cdot\frac{\partial\pi_1}{\partial n}+c_3\,\delta\cdot\frac{\partial\theta_1}{\partial n}=\frac{3}{2}(\mathrm{E}+\mathrm{M})\,\delta\cdot\frac{\partial}{\partial n}\left(\frac{1}{r}\right)_0,$$

$$c_2\,\delta\cdot\frac{\partial\pi_1}{\partial\cdot e^2}+c_3\,\delta\cdot\frac{\partial\theta_1}{\partial\cdot e^2}=\frac{3}{2}(\mathrm{E}+\mathrm{M})\,\delta\cdot\frac{\partial}{\partial\cdot e^2}\left(\frac{1}{r}\right)_0,$$

$$c_2\,\delta\cdot\frac{\partial\pi_1}{\partial\cdot\gamma^2}+c_3\,\delta\cdot\frac{\partial\theta_1}{\partial\cdot\gamma^2}=\frac{3}{2}(\mathrm{E}+\mathrm{M})\,\delta\cdot\frac{\partial}{\partial\cdot\gamma^2}\left(\frac{1}{r}\right)_0.$$

These results immediately follow from those given in my former paper in the *Monthly Notices* by submitting them to a variation δ and putting δc_1, δc_2, δc_3 zero. As before, $\mathrm{E}+\mathrm{M}=n^2a^3$, and $\left(\frac{1}{r}\right)_0$ denotes the constant term in the expression of $\frac{1}{r}$; the constants are defined as in Delaunay's final expressions for the coordinates. The square of the ratio of the solar to the lunar parallax is neglected, but its effect is quite insensible.

The first of these three equations may be used effectively to find the secular accelerations. We have

$$\delta\cdot\frac{\partial\pi_1}{\partial n}=\frac{\partial^2\pi_1}{\partial n^2}\,\delta n+\frac{\partial^2\pi_1}{\partial n\,\partial\cdot e'^2}\,\delta\cdot e'^2+\frac{\partial^2\pi_1}{\partial n\,\partial\cdot e^2}\,\delta\cdot e^2+\frac{\partial^2\pi_1}{\partial n\,\partial\cdot\gamma^2}\,\delta\cdot\gamma^2,$$

with similar results for the other expressions. It can be easily shown that c_2, $\delta\cdot e^2$ contain the factor e^2 in all their terms, and c_3, $\delta\cdot\gamma^2$ the factor γ^2. Also it is known by a theorem of Adams * that there are no terms in $\left(\frac{1}{r}\right)_0$ of the form $\frac{1}{a}e^2 f(m,e'^2)$, $\frac{1}{a}\gamma^2 f(m,e'^2)$. Hence, in order to find δn to the orders e^2, γ^2, and all powers of e'^2, we can neglect $\delta\,.\,e^2$, $\delta\,.\,\gamma^2$ in the first equation, so that it becomes

$$c_2\left(\frac{\partial^2\pi_1}{\partial n^2}\,\delta n+\frac{\partial^2\pi_1}{\partial n\,\partial\,.\,e'^2}\,\delta\,.\,e'^2\right)+c_3\left(\frac{\partial^2\theta_1}{\partial n^2}\,\delta n+\frac{\partial^2\theta_1}{\partial n\,\partial\,.\,e'^2}\,\delta\,.\,e'^2\right)$$

$$=\frac{3}{2}(\mathrm{E}+\mathrm{M})\left[\frac{\partial^2\mathrm{P}_0}{\partial n^2}\,\delta n+\frac{\partial^2\mathrm{P}_0}{\partial n\,\partial\,.\,e'^2}\,\delta e'^2\right],$$

where P_0 has been put for $\left(\frac{1}{r}\right)_0$.

If we neglect e^2, γ^2, it is seen that the principal part of δn is given by

$$\frac{\partial^2\mathrm{P}_0}{\partial n^2}\,\delta n=-\frac{\partial^2\mathrm{P}_0}{\partial n\,\partial\,.\,e'^2}\,\delta e'^2\quad\ldots\ldots\quad(1)$$

* *Monthly Notices*, vol. xxxviii. pp. 460–472. *Coll. Works*, vol. i. pp. 189–204.

When the principal part of δn has been found from this equation, its value may be substituted in the left-hand member of the previous equation, and the parts depending on e^2, γ^2 can then be evaluated.

For the motion of the perigee we have

$$\delta\pi_1 = \frac{\partial \pi_1}{\partial \cdot e'^2}\delta \cdot e'^2 + \frac{\partial \pi_1}{\partial n}\delta n + \frac{\partial \pi_1}{\partial \cdot e^2}\delta \cdot e^2 + \frac{\partial \pi_1}{\partial \cdot \gamma^2}\delta \cdot \gamma^2.$$

If we neglect e^2, γ^2, it will be sufficient to use the first two terms only. The parts of chief importance in all cases arise from $\partial\pi_1/\partial \cdot e'^2$, and their accuracy depends mainly on the accuracy with which the terms having the factor e'^2 in π_1 are known. If, however, the parts in $\delta\pi_1$, depending on e^2, γ^2, be required, it will be simplest to find $\delta \cdot e^2$, $\delta \cdot \gamma^2$ from the equations $\delta c_2 = 0$, $\delta c_3 = 0$, respectively. The principal parts of $\delta \cdot e^2$, $\delta \cdot \gamma^2$ are by this method found in the forms

$$n\delta \cdot e^2 = e^2 f(m)\delta n, \qquad n\delta \cdot \gamma^2 = \gamma^2 f(m)\delta n,$$

and can therefore be found, since the value of δn is already known.

In the present state of the lunar theory nothing is to be gained by calculating these higher portions in the case of the perigee, for the complete numerical values of the terms in π_1, which have the factors $e^2e'^2$, $\gamma^2e'^2$ have not yet been obtained, and Adams' value of P_0 furnishes only the terms up to the orders $e^2c'^2m^3$, $\gamma^2e'^2m^3$. Hence we must verify Delaunay's values to this order and estimate for the remainders. Fortunately, owing to the smallness of the quantities involved, the possible errors of estimation are very small and can be easily seen.

Exactly similar results hold for $\delta\theta_1$.

3. The equation (1) explains partly the slowness of the convergence of the expression which represents the principal part of δn when expanded in powers of m. All inequalities in the lunar theory which contain the factor e' appear to be more or less slowly convergent. When such expressions as $e'^2f(m)$ are differentiated with respect to n, the earlier terms will converge more slowly still, but the convergence of the later terms will not be much affected. The calculations further show that slow convergence in the higher terms will arise partly from the expansion of the inverse of $\frac{\partial^2 P_0}{\partial n^2}$ in powers of m ; a remedy for this is evident. The convergence of all the results is, as usual, improved by using $m_1 = n'/(n-n')$ instead of $m = n'/n$ as a parameter.

4. *A Test of Delaunay's Expressions for the Secular Accelerations.*—These are given in *C.R.* vol. xliv. p. 209, in terms of Plana's constants, and, with an increased number of terms, in *C.R.* vol. lxxiv. p. 174, in terms of his own constants. The constant part

of $1/r$ has been obtained by Adams (*Coll. Works*, vol. i. p. 203), and his value was used in the formulæ of § 2. The values of c_2, c_3 are given by Newcomb (mem. cit. p. 184), and the values of π_1, θ_1 are those of Delaunay (*C.R.* vol. lxxiv. p. 19) with the corrections noted in my paper in the *Monthly Notices* already referred to (§ 1). The last powers of m tested in each case are as follows :—

The value of $\delta\epsilon$ was tested as far as m^7, γ^2m^5, e^2m^5, e'^2m^5, and the values of $\delta\pi$, $\delta\theta$ as far as m^5, γ^2m^3, e^2m^3, e'^2m^2. The only differences found were the coefficients of the terms involving γ^2m^3 in $\delta\pi$ and e^2m^3 in $\delta\theta$, for which $-\frac{694}{4}$, $-\frac{694}{16}$ were found instead of $-\frac{693}{4}$, $-\frac{693}{16}$. The terms involving a^2/a'^2 and higher powers and products of e^2, γ^2, e'^2 do not produce additions so great as $0''{\cdot}01$, and they will therefore not be considered.

5. *Method of combining algebraical and numerical results to obtain the derivatives with regard to* n.—The problem may be stated as follows : Given the first i terms and also the complete numerical value of a series whose terms are diminishing in absolute value, to find a close approximation to the value of its derivative. The following is a practical method of solution :—

Let

$$f(m) = \alpha_0 + \alpha_1 m + \alpha_2 m^2 + \ldots,$$

where α_0, α_1, . . are numerical coefficients, and put

$$f_i(m) = \alpha_0 + \alpha_1 m + \alpha_2 m^2 + \ldots + \alpha_{i-1} m^{i-1}.$$

Taking the derivatives with regard to m, we can write the result

$$mf'(m) = mf_i'(m) + i\left[\alpha_i m^i + \frac{i+1}{i}\alpha_{i+1}m^{i+1} + \ldots\right].$$

If i be not too small (*e.g.* not less than 5), and the terms of the series are not diminishing too slowly (*e.g.* with a ratio not much greater than $\frac{1}{2}$), the error committed by putting

$$mf'(m) = mf_i'(m) + i[mf(m) - mf_i(m)]$$

will be small compared with the true value of $mf'(m)$. We shall generally obtain greater accuracy by using $i+$ a proper fraction instead of i as the factor, when the series appears to be diminishing with fair regularity.

For example, the part of π_1 which depends on e'^2 when the parameter m_1 is used, is

$$ne'^2\left(\frac{9}{8}m_1^2 + \frac{753}{32}m_1^3 + \frac{42243}{256}m_1^4 + \frac{942777}{1024}m_1^5\right).$$

Here $i=6$, and reducing the portion in brackets to numbers, we obtain

$$f_6(m_1) = +\cdot00735 + \cdot01244 + \cdot00705 + \cdot00318 = +\cdot03002.$$

I find

$$f(m_1) = +\cdot032063.$$

Hence

$$f(m_1) - f_6(m_1) = +\cdot00204.$$

Applying the formula, and using $6\frac{1}{2}$ as the multiplying factor of this difference, we find

$$f'(m_1) = +\cdot0147 + \cdot0373 + \cdot0282 + \cdot0159 + \cdot0133 = +\cdot1094.$$

The result might possibly be as much as four or five units wrong in the last place of decimals, but in all the cases treated below, the possible error does not affect the results to the degree of accuracy required. To complete the determination, we have, for this portion of π_1,

$$\frac{\partial \pi_1}{\partial n} = f(m_1) - (1 + m_1) m_1 f'(m_1)$$
$$= +\cdot0321 - \cdot1182 = -\cdot086,$$

which is not more than one unit wrong in the third place of decimals.

The Numerical Values.

6. The following are the numerical data required :—

In the values of c_2, c_3, we only require the terms which are of the forms $na^2e^2f(m)$, $na^2\gamma^2f(m)$, respectively. These may be accurately obtained by Newcomb's method from the expressions in terms of rectangular coordinates, c_3 from the results in a paper by Mr. P. H. Cowell,* and c_2 from those in a paper by the writer.† We find

$$c_2 = -\cdot474035na^2e^2, \qquad c_3 = -2\cdot00257na^2\gamma^2.$$

The values of π_1, θ_1 are as follows :‡

$$\pi_1 = n[+\cdot0085726 + \cdot032063e'^2],$$
$$\theta_1 = n[-\cdot0039992 - \cdot005224e'^2].$$

Proceeding as in § 5, we then find

$$\frac{\partial \pi_1}{\partial n} = -\cdot01502 - \cdot086e'^2,$$

$$\frac{\partial \theta_1}{\partial n} = +\cdot03708 + \cdot00319e'^2,$$

$$n\frac{\partial^2 \pi_1}{\partial n^2} = -\cdot0489, \qquad n\frac{\partial^2 \theta_1}{\partial n^2} = -\cdot00661.$$

* *Amer. Jour. Math.* vol. xviii. p. 113.

† *Ibid.* vol. xv. p. 261.

‡ See the paper "On the Mean Motions of the Lunar Perigee and Node."

The numbers contain powers of m only ; the omitted portions, depending on e^2, etc., are not required.

Further, from Adams' algebraical results and my numerical calculations :

$$aP_0 = 1{\cdot}00091 + {\cdot}0011750e'^2 + {\cdot}00064e'^4,$$

$$an\frac{\partial P_0}{\partial n} = +{\cdot}66551 - {\cdot}0009865e'^2 + {\cdot}0026e'^4,$$

$$an^2\frac{\partial P_0}{\partial n^2} = -{\cdot}21975 - {\cdot}0001315e'^2.$$

7. Pursuing the operations of § 2 above, we obtain

$$\delta n = [-{\cdot}004489 + {\cdot}024e'^2 + {\cdot}1235e^2 - {\cdot}0196\gamma^2]n\delta \,.\, e'^2,$$
$$\delta\pi_1 = +{\cdot}03213n\delta \,.\, e'^2,$$
$$\delta\theta_1 = -{\cdot}005241n\delta \,.\, e'^2.$$

Put now, with Delaunay,

$$\int n\delta e'^2 dt = -1270''\mathrm{T}^2,$$

where T is the time reckoned in centuries of Julian years, and include the parts of $\delta\pi_1$, $\delta\theta_1$ which depend on e^2, γ^2, e'^2 from his expressions (see § 4 above), estimating for the remainders in each case. The maximum possible error due to the latter is appended, with the resulting values in the table :

Secular Accelerations.

Parts of form.	In δn.	In $\delta\pi_1$.	In $\delta\theta_1$.
$f(m)$	$+5''{\cdot}70$	$-40''{\cdot}8$	$+6''{\cdot}66$
$e'^2 f(m)$	$-0{\cdot}01$	$+\ 0{\cdot}0$	$-0{\cdot}00$
$e^2 f(m)$	$+0{\cdot}47$	$+\ 0{\cdot}2$	$+0{\cdot}14$
$\gamma^2 f(m)$	$-0{\cdot}05$	$+\ 0{\cdot}4$	$-0{\cdot}02$
Complete values	$+6{\cdot}11 \pm 0''{\cdot}02$	$-40{\cdot}2 \pm 0''{\cdot}1$	$+6{\cdot}78 \pm 0''{\cdot}02$

These results agree almost exactly with those of Delaunay. With the latest values for the masses of the planets, Professor Newcomb finds (mem. cit. p. 199),

$$\int n\delta e'^2 = -1229''\mathrm{T}^2.$$

Hence the definitive theoretical values of the secular accelerations per century are, for

The Mean Motion	...	...	...	$+\ 5''{\cdot}91 \pm 0''{\cdot}02$;
The Perigee	...	...	...	$-38{\cdot}9\ \pm 0{\cdot}1$;
The Node	...	...	...	$+\ 6{\cdot}56 \pm 0{\cdot}02$.

Laying aside the question of the accuracy of the value of $\int n\delta e'^2$, the above theoretical values for the secular acceleration are correct to $\frac{1}{300}$ part of the whole in each case.

Haverford College, Pa.:
1897 *January* 11.

On a Photographic Transit Circle. By H. H. Turner, M.A., B.Sc., Savilian Professor.

1. The desirability of introducing photography into meridian observations, as a method of eliminating personal and other errors, has been recognised by many astronomers, and will probably be admitted by all, if this introduction can be accomplished without endangering the simplicity and stability of the present form of instrument. The following solution of this problem, which presents some advantages not realised in other forms, is offered for consideration.

2. It may be well in the first instance to enumerate the chief features of the present instrument—its advantages and disadvantages.

Features:

(*a*) Movable visual telescope with two micrometers at right angles.

(*b*) Two carefully made pivots, allowing one degree of freedom only.

(*c*) Graduated circle and microscopes.

(*d*) Chronograph.

All these, or their equivalents, may thus be allowed in any new arrangement.

Advantages:

(*e*) Stability, owing to there being only one degree of freedom, excepting in so far as the micrometers are used.

(*f*) Simplicity.

The advantages may generally be brought under one or other of these heads. It is unnecessary to specify them in detail unless we are going to propose any rearrangement of the instrument which will sacrifice any of them, and that is not my present purpose. It is more important to consider carefully how we may remove the

Disadvantages of the Present Instrument, viz.:

(*g*) Personal equation of all kinds.

(*h*) Flexure and irregular refraction.

(*i*) Limitation to comparatively bright stars.

3. Considering first (*g*), personal equation, photography is clearly useful here. In measuring a photograph different persons usually have a small 'error of bisection,' but this can be eliminated at leisure by turning the photograph round and remeasuring it, and a successful photographic method would thus be of the greatest advantage, not only in longitude work, where the absolute time is required, but in forming star catalogues, which are at present affected with the personality dependent on brightness of the star.

4. With regard to (*h*) I have already proposed (*Monthly Notices*, vol. liv. p. 412) an arrangement of a fixed telescope pointing to a plane mirror. The mirror can be made thick enough to be free from flexure without being nearly so heavy as a large telescope, and if carefully mounted in the meridian, in the same way as a transit is mounted, nothing would be lost of stability and very little of simplicity. The graduated circle (or equivalent) would now be attached to the mirror. It was further pointed out in the paper above cited that the effects of irregular refraction from the atmosphere of the observing room (small and much over-rated, as I believe these to be, see *Mem. R.A.S.* vol. li. pp. 110–116 and *Monthly Notices*, vol. liv. p. 488) are minimised by this arrangement, for the mirror may be placed practically in the open air at a distance from the observer, so that the light of a star travels as far as the mirror without much disturbance from irregular strata of air. On their way from the mirror to the observer the rays of light may suffer such disturbance, but this will be the same for all stars, whatever the zenith distance, and hence will go out in the zenith point.

5. With regard to (*i*), it is true that observations of stars and small planets to about the 10th magnitude are found possible at (*e.g.*) the Royal Observatory, Greenwich, with the transit circle of 8-inch aperture. But even then, for the fainter objects, illuminated wires must be used instead of the illuminated field, and the observations are found to lose in accuracy. With smaller instruments the range of available stars is more restricted, and it will not be possible for us at Oxford (with the Barclay transit circle of 4 in. aperture, and with no apparatus for illuminating the wires) to reobserve the great majority of the faint stars of the *Cambridge Catalogue* (1875·0), which we are using as reference stars for the work of the *Astrographic Catalogue*.

But one of the chief uses of photography is to bring faint objects within easier reach, and if we can use a fairly long exposure (of some minutes, say) there is no reason why an instrument of moderate power should not photograph stars to the 9th or 10th magnitude. We see at once, however, that to use photographic trails will not do; we are just as much restricted as before to bright stars. Something must move to follow the star, and that cannot be the telescope which we have already fixed, nor the mirror which is to have only one degree of freedom. Hence it must be the photographic plate. As the star travels

across the field of the fixed telescope, which we may assume about 2° in breadth, the plate must be moved so that the star falls continuously on one spot, which thus receives an exposure of 8 minutes at the equator. This should be sufficient to photograph tolerably faint stars with a good photographic lens of modest dimensions. In any case, those observatories which possess instruments for the work of the Astrographic Chart would be able at some time to remove the object glass in its cell and mount it in a wooden tube, which would be all that is necessary in the case of a fixed telescope ; and then an exposure of 8 minutes at the focus would give 11th or 12th magnitude stars.

6. Now it looks at first sight as though moving the plate were just as bad as moving the telescope or the mirror, were, in fact, giving up the idea of having only one degree of freedom. But by a simple device we can entirely obviate this objection. The plate need not be fixed to the telescope ; all that is necessary is that *the position of the plate with reference to the fixed telescope at any instant should be recorded.* Suppose a spot of light from a fixed source to shine on the plate every second, the interruptions being made by the standard clock. As the plate travels across the field a series of dots will be formed which will show the irregularities of the motion. If there be two spots (one at the top and one at the bottom, say) we shall know the orientation of the plate as well as its position in the slide at any instant, and if the motion be tolerably uniform, such as can be secured by any one of a number of mechanical devices, the position of any star photographed can be referred to the mean of these seconds' spots on the plate. The process will be rather like referring a star to the several wires of the transit reticule, in fact the moving plate is now our chronograph, so that we are not really adding anything to the "features" specified in § 2. In the case of bright stars, the exposure would only be a few seconds, and the measures of the spots could be quickly made. In the case of longer exposures, of 8 minutes, say, it would probably not be necessary to measure the whole $8 \times 60 = 480$ spots ; if every 10th or even 30th were measured, enough information would probably be obtained as to the regularity of the clock driving to show whether it was necessary to proceed further.

7. Before entering further into the details of the arrangement let us now sum up the features of the new instrument, and compare them with the ordinary transit circle. The following is a brief tabular comparison of this kind :—

Ordinary Transit Circle.	*Proposed Photographic Transit Circle.*
(*a*) Movable visual telescope with two micrometers at right angles.	(*a*) Fixed photographic telescope. The micrometers are not attached to the telescope, but are used to measure the photographic plate.

(*b*) Two carefully made pivots, allowing one degree of freedom only.	(*b*) The pivots attached to a plane mirror, which has thus one degree of freedom only.
(*c*) Graduated circle and microscopes.	(*c*) Retained, in connection with the mirror.
(*d*) Chronograph.	(*d*) A corresponding arrangement moves and records position of the photographic plate.
(*e*) Stability.	(*e*) Retained as in (*b*).
(*f*) Simplicity.	(*f*) The only additional apparatus is the plane mirror.
(*g*) Personal equation affects the observations.	(*g*) Personal equation minimised or eliminated.
(*h*) Flexure and irregular refraction.	(*h*) The flexure of a comparatively small mirror substituted for that of a long tube held by the middle. Effects of irregular refraction minimised, the mirror being in the open.
(*i*) Limitation to bright stars.	(*i*) Stars of 10th magnitude or fainter within reach.

8. This theoretical comparison seems to promise well, and I see no reason why it should break down in practice. There are, of course, numerous small points of detail to be worked out, but none of them present any great difficulties. Most of them, though perhaps not all, are considered in the following paragraphs. I am proceeding with the mounting of an instrument at the University Observatory, and a fuller account will be given after practical experience of the working.

9. *Movement of the Plate.*—So long as the driving is regular it is not necessary that it be *exactly* of the stellar rate unless we want a very faint star. If the stellar rate and the plate rate differ the star will trail slightly, but we shall make no sensible error in referring the middle of the trail to the mean of the seconds' marks.

10. Now stars of different declinations travel at different rates ; hence we must have some contrivance for adjusting the rate at which the plate travels. Various devices might be used. There is an ingenious one already in use at Greenwich for the personal equation machine (*Monthly Notices*, vol. xlviii. p. 3) ; but perhaps the simplest would be to have a long graduated arm, pivoted at one end, the other moved by the clock ; a string attached to the plate holder and passing over a fixed pulley (so that the pull on the plate is always in the same direction) could be attached to this arm at any distance from the pivot, and would thus pull the plate along at a suitable rate. But the details are still to be worked out.

11. *The Variable Chronographic Scale.*—For stars near the Pole the seconds' marks would be close together, while for

equatorial stars they would be better separated. The chronograph scale is thus wider for the latter than the former, and given errors of measurement affect the time of transit less in the ratio of the secant of declination. But it is easily seen that the error of transit in arc of a great circle is the same for all stars. There is, however, another inconvenience in a close scale, viz. that the spots would interfere with one another. This must be obviated by cutting out some of them. For instance, the observer might by hand hold down the electro-magnet or relay, &c. which admits the light periodically, only allowing it to rise every fifth or tenth second, as the case may be.

12. *Rotation of Plate.*—Another point to be considered is the rotation of the field of view, and the differential motion for stars in the same field, for one great advantage of this form of instrument is the possibility of photographing (and reducing together) a number of stars on the same plate. There are various mechanical contrivances by which a proper amount of rotation as well as of translation could be given to the plate ; for the present I describe the simplest merely. Suppose for a moment the instrument is to be devoted to the reobservation of the (faint) stars in zone $+25°$. The plates taken with it will be of two kinds.

(A) Those for adjustments, for which bright stars may be used, though of all declinations.

(B) Those in zone $+25°$, which are faint.

Now construct a wooden arc of such a radius that the plate-holder will move along it so as just to suit zone $+25°$, and determine very accurately the point of the driving-arm which suits this exact declination, so that we get nice round discs for the stars in this zone readily. For other stars let the plate move along the same arc, but alter the rate to suit the star, though perhaps a rough approximation will suffice. The stars will trail slightly, but the exposures can be short, and no sensible error will be introduced. Indeed, we need not perhaps alter the rate of motion for some of the bright stars, but simply allow them to trail. When zone $+25°$ has been well observed we can make another arc for zone $+26°$ or any other zone. But if it be required to take faint stars in different zones on the same night we must make some linkage or epicycloidal arrangement for moving the plate.

13. *The Fixed Spots of Light.*—With regard to the spot or spots of light which mark the plate, there are advantages in the following arrangement : Let there be a point-source of light in the plane of the photographic film, but just beyond the field, above or below ; which sends rays through the objective of the telescope. When the mirror is nearly normal to the telescope axis it will reflect these rays back through the objective to a point on the film (supposed at rest), which will become a line if the mirror is rotated gently. This line (l_1) obviously furnishes an important adjustment ; if it is perfectly straight, the plane of the mirror is parallel to the horizontal axis, and if the point-source lies in its prolongation, the telescope is perpendicular to

the mirror. Further, if a mercury trough be placed below the mirror, and the mirror turned into a suitable position, another point or line (l_2) will be obtained showing the level adjustment of the horizontal axis. Now let there be a small, *fixed* mirror nearly normal to the telescope axis, which constantly reflects *a portion* of the light from the point-source on to the plate, at a point (l_3). The reflection from this mirror is to be the chronograph index, and is to be suitably interrupted by the clock. The distances l_3–l_1 and l_3–l_2, which can be recorded and measured whenever necessary, will furnish the necessary constants for reduction of the observations for collimation, level, and index error. The small fixed mirror will necessarily come between the objective and the large mirror, and will thus block out a portion of the objective ; it should thus be as small as possible. In some cases a small circular patch of tin-foil, placed on the outer surface of the object glass, might serve the purpose. As already remarked, it will be better to have *two* point-sources of light, one above and one below, to check the orientation of the plate.

14. *Collimation of the Instrument.*—The method of determining this has been partly indicated in the last paragraph. There are three points to be considered :—

(α) The mirror must be placed parallel to its axis of rotation.

(β) We must determine where a plane through the centre of the objective perpendicular to the mirror axis cuts the plate.

(γ) We must determine the positions of the point-source and its reflected images with reference to the plane specified in (β).

As regards (α), we have always the expedient of reversing the mirror in its Ys, and comparing the images either of the spot of light or of a star.

As regards (β) and (γ), enough has been said in the last paragraph.

It may be remarked that the *correction* for collimation error with this instrument need not be made in the familiar way, but can be applied as an index error to the measures of position on the plate. Suppose the meridian on the celestial sphere were a continuous bright line, and that the axis of the mirror were truly horizontal E. and W. (*i.e.*, no level and azimuth errors), but the mirror inclined to it at any angle. Then, whatever be the position of the mirror, the image of the meridian on the photographic plate would be the same curved line, part of a small circle parallel to the meridian and at an angular distance from it equal to twice the tilt of the mirror. A star would be on the meridian when its image fell on this fixed line, the position of which can be determined with reference to the seconds' marks on the plate.

15. *Level Error.*—A striding level may be applied to the pivots of the mirror in the usual way, or the method of reflection from mercury indicated in § 13 may be used. Or stars may be observed by reflection from mercury.

16. *Azimuth Error and Clock Error* are found from stars in the usual way.

A Photographic Zenith Tube.

17. The general principle of a moving photographic plate, which is rendered equivalent to a fixed plate by the marking of a fixed index, would appear to be advantageously applicable to the zenith tube, especially the floating zenith tube. The actual tube is not of much consequence, since we are not using it to bind object-glass and plate, and may be dispensed with altogether, or detached from the objective and used as a shield from stray-light merely. The important point is to have the index which marks the plate rigidly attached to the floating objective. For the present it is not necessary to specify the way in which the marking of the plate shall be accomplished ; but let us suppose some lamp, or lamp and prism, or other source of light, firmly attached to the floating objective, and so arranged that it makes a spot of light on a plate placed in the focus of the objective, and first suppose this plate fixed. Further assume that there are suitable arrangements for rotating the objective and its index appliance in any azimuth.

18. Now suppose a star at rest in the zenith. On exposing the plate it will make a spot on the film ; and a second spot will be made by the index—let us say at distance d and in azimuth ϕ. Rotate the objective through 180°. If the centre of the objective remains accurately in the same place the star will focus on exactly the same spot as before ; and the index-mark will be at the same distance, d, from it in azimuth $\phi + 180°$. If, however, the objective is shifted laterally in the process of rotating it (which would, indeed, generally be the case in such an arrangement), the star image would no longer fall on the same spot as before, but would deviate from it by the amount of the lateral shift. But the index-spot would be displaced in exactly the same manner, and hence it would still be in the same position relatively to the star image—viz., at distance d and in azimuth $180° + \phi$. (In fact a lateral shift of the objective and index together is equivalent to a movement of the photographic plate, which we are going to admit freely.) Thus the mean positions of the index and the zenith coincide ; and, similarly, if the star is not actually in the zenith, but at some distance from it, we obtain its zenith distance from two photographs in reversed positions.

19. In symbols let x and y be the rectangular coordinates of the star referred to the index dot, measured in any fixed directions ; and let their values for the first exposure be x_1, y_1, and for the second x_2, y_2. Then

$$x_1 = +d \cos \phi \qquad y_1 = +d \sin \phi$$
$$x_2 = -d \cos \phi \qquad y_2 = -d \sin \phi$$

Here we assume that the plate is rotated through exactly

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180°. There will probably be a slight error in doing this—say $\Delta\phi$. Then for the second pair of equations we must write

$$x_2 = -d\ (\cos\phi - \sin\phi\Delta\phi) \qquad y_2 = -d\ (\sin\phi + \cos\phi\ .\ \Delta\phi).$$

Thus

$$(x_1 + x_2) = d\sin\phi\ .\ \Delta\phi \qquad y_1 + y_2 = -d\cos\phi\ .\ \Delta\phi.$$

We can thus reduce the effect of error of orientation $\Delta\phi$ on the x coordinate to an insensible quantity by making $d\sin\phi$ small, and on the y coordinate by making $d\cos\phi$ small. But $d\cos\phi$ and $d\sin\phi$ are the coordinates of the index spot with respect to the zenith. Hence we must adjust the spot to be as nearly as possible at the zenith. Or two spots at equal zenith distances would do equally well, and would check the orientation of the plate.

20. Now, as in the case of the transit-circle, let the plate move so as to follow the image of the star, and get the benefit of a prolonged exposure, so that we may photograph faint stars in the zenith; and let the index-spot be recorded every second only. We can determine their R.A.'s and zenith distances by referring the stars to the index-spots. Whenever a sufficiently bright star is passing near the zenith, we may reverse the instrument during the transit, obtaining two images of the same star for determination of index error; or we may reverse between the transits of two stars whose relative places are known; or again, we may reverse in opposite senses on two different nights between the transits of any two stars known or unknown.

21. Almost the only adjustment required is the centring of the index-spot. The only constant required is the scale value in arc on the plate, which is shewn by the scale of seconds marks when the star does not trail. In addition the plate should be horizontal; the bed on which it moves can be probably levelled with sufficient accuracy by mechanical means, or the plate might be floated on mercury.

22. If a series of plates, or perhaps a long film, be arranged on a slide, or on a rotating horizontal tray, so that each in turn is presented for exposure, the whole zone of stars which passes the zenith might be photographed every clear night with comparatively little labour. An occasional reversal of the object-glass would give the constant of reduction.

23. *Other Applications.*—The principle of a moving plate, marked by a fixed index-spot, is applicable generally to the case of any instrument for determining positions. As another example we may take the fixed camera proposed by Capt. E. H. Hills, R.E. (*Monthly Notices*, vol. lv. p. 89), for determining longitudes by photography; an arrangement which may also be used for determining the place of the Moon. So long as we fix the plate to the camera, we are limited to bright stars; but we can extend the observations to faint stars by moving the plate as above.

University Observatory, Oxford:
1897 *March* 5.

The Sidereal System Revised in 1896. By Maxwell Hall.

The original article (*Memoirs R.A.S.* vol. xliii. p. 157) was revised in 1887 (*Monthly Notices*, vol. xlvii. p. 521); the present revision is undertaken to meet the determination by photography at the Potsdam Physical Observatory of the radial velocities of fifty-one stars, the determination by photography at the Oxford University Observatory of the annual parallaxes of about twenty-six stars of the second magnitude conveniently situated in that latitude, and to meet the recognition of the greater accuracy of the stellar proper motions as deduced by Dr. Auwers.

In the present revision the latter proper motions are always used when possible.

Note I. contains a list of those stars whose proper motions have been taken from other authorities.

Note II. contains a list of observed parallaxes; the means have been adopted.

Note III. contains the radial velocities observed by Dr. Huggins, at Greenwich, and at Potsdam. It is disappointing to find so many contradictory results; in the case of many stars the adopted means seem very uncertain.

It will be noticed that the general effect of the Potsdam observations is to greatly reduce all radial velocities; and it will be found that the Oxford parallaxes are much larger than the computed parallaxes; consequently it appears that there are large errors among the constants of the system hitherto adopted; and these large errors must first be ascertained before we can possibly proceed.

From the revision of 1887 the value of ΠV was found to be $0''\cdot10$; and from the manner in which ΠV and V enter the computations it will be seen that if ΠV be still considered $0''\cdot10$, but that if V be reduced to 5 and Π increased to $0''\cdot02$, then all the radial velocities will be diminished about one-half, and all the parallaxes will be nearly doubled.

Let us consider the effect of these changes on the very few stars whose parallaxes and radial velocities have both been ascertained with considerable accuracy:—

Star.	Observed. ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$
	"		"	
Sirius	0·27	0?	0·30	0
Arcturus	0·07	− 9	0·38	− 7
Capella	0·07	+ 7	0·06	+ 4
Rigel	0·03	+ 4	0·03	+ 9
Vega	0·12	− 9	0·16	−10
Procyon	0·23	+ 2	0·14	+ 4
α Orionis	0·02	+ 6	0·03	+ 7
Altair	0·20	− 8	0·32	− 7
Aldebaran	0·10	+11	0·04	+ 7
Polaris	0·05	− 6	0·02	− 6

Among these twenty results of computation the only serious difference between observation and computation occurs in the case of the parallax of *Arcturus*. Peters obtained 0″·127, Elkin 0″·016, so that the mean 0″·07 must yet be considered uncertain.

Omitting, then, the parallax of this star, the sums of the observed and computed parallaxes agree; and the sums of the observed and computed radial velocities, neglecting their signs, also agree.

In the following general computations there will be considerable saving of labour by employing only three decimal places for both natural numbers and logarithms instead of five, as before. In the case of any star the results depend upon its position and proper motion; and although the functions depending on position may be computed with accuracy, yet three places of decimals will clearly give as much numerical accuracy to the functions depending on proper motion as the uncertainty connected with proper motion will justify.

It was shown in the last revision that when failure occurred in computation such failure was generally due to the assumption of circularity of the star's orbit, and elliptic orbits were discussed.* We shall here give the cubic equation for u, so that in difficult cases it may be easily ascertained whether such failure exists or not—indeed, in cases where ρ_0 is not approximately known, much time may be saved by its use.

The cubic equation for u. $[v^2 = \mathrm{F}r_0^2.]$

$$\text{We have } u = \frac{\mathrm{K} + a\Pi\mathrm{V}}{\mathrm{H} + \Pi\rho_0},$$

$$\text{and } \rho_0 = \frac{2\mathrm{V}\,(au - b + \mathrm{H}\Pi\mathrm{V})}{c^2 + u^2 - \Pi^2\mathrm{V}^2}.$$

* In the fig. *Monthly Notices*, vol. xlvii. p. 522, *dele* the letter ψ; and *dele* the words "the angle ACS," p. 523, line 8.

Multiply the last equation by Π, and add H to both sides, then

$$H + \Pi\rho_0 = \frac{2\Pi V(au - b + H\Pi V) + H(c^2 + u^2 - \Pi^2V^2)}{c^2 + u^2 - \Pi^2V^2}.$$

Invert, and multiply by $(K + a\Pi V)$; then

$$u = \frac{(K + a\Pi V)(c^2 + u^2 - \Pi^2V^2)}{2\Pi V(au - b + H\Pi V) + H(c^2 + u^2 - \Pi^2V^2)}.$$

Let

$$A = \frac{K - a\Pi V}{H};$$

$$B = c^2 + \Pi^2V^2 - \frac{2b}{H}\Pi V;$$

$$C = \frac{K + a\Pi V}{H}(c^2 - \Pi^2V^2);$$

then

$$u^3 - Au^2 + Bu - C = 0.$$

This equation may easily be solved by approximation, and then

$$\rho_0 = \frac{K + a\Pi V - Hu}{\Pi u}.$$

Up to the present we have assumed that the stars are uniformly distributed, so that the force to the centre varies directly as r_0, and so that in circular orbits

$$v^2 = Fr_0^2.$$

But from considerations based upon the number and magnitude of the stars there is good reason to suppose that the number of the stars in an unit volume of space, or, in other words, that the star-density of space, varies inversely as the distance from the centre.

In this case the force to the centre is constant, and for circular orbits

$$v^2 = Fr_0.$$

Now this change in the law of force has little or no effect on the nearer stars; and in order to ascertain which law of force gives the better results we shall have to compute the parallaxes and radial velocities of the stars upon the two suppositions.

It will be convenient to collect all the formulæ for computation:

θ = North Polar Distance
ϕ = Right Ascension
} of a star, Jan. 1·0, 1850;

$\delta\theta$ = Annual proper motion in θ
$\delta\phi$ = Annual proper motion in ϕ
} in seconds of arc;

ϖ = Annual parallax, in seconds of arc;

ρ_0 = Distance in sidereal units = $\frac{1}{\varpi}$;

$\frac{d\rho}{dt}$ = Annual change in distance, taking the radius of the Earth's orbit as the unit of space.

$$l = \sin\theta\cos\phi\,; \quad \delta l = n\delta\theta\,.\cos\phi - m\delta\phi\,;$$
$$m = \sin\theta\sin\phi\,; \quad \delta m = n\delta\theta\,.\sin\phi + l\delta\phi\,;$$
$$n = \cos\theta\,; \quad \delta n = -\sin\theta\,.\,\delta\theta.$$

$$a = l\lambda + m\mu + n\nu\,; \qquad \log\lambda = -9{\cdot}166$$
$$b = \lambda\delta l + \mu\delta m + \nu\delta n\,; \quad \log\mu = -9{\cdot}914$$
$$c^2 = \delta l^2 + \delta m^2 + \delta n^2\,; \quad \log\nu = +9{\cdot}744.$$

$$H = lL + mM + nN\,; \qquad \log L = -9{\cdot}936$$
$$K = L\delta l + M\delta m + N\delta n\,; \quad \log M = -9{\cdot}223$$
$$\log N = -9{\cdot}678.*$$

$$V = 5\,; \quad \Pi V = 0{\cdot}100\,; \quad \Pi = 0{\cdot}020.$$

$$u = \frac{K + a\Pi V}{H + \Pi\rho_0}\,; \quad \frac{d\rho}{at} = -u\rho_0.$$

Then for $v^2 = Fr_0^{-2}$ we have

$$\rho_0 = \frac{2V(au - b + H\Pi V}{c^2 + u^2 - \Pi^2V^2}.$$

And for $v^2 = Fr_0$ we have

$$\rho_0 = \frac{2V\left(au - b + \frac{H\Pi V}{1 + \Pi r_0}\right)}{c^2 + u^2 - \frac{\Pi^2V^2}{1 + \Pi r_0}}$$

where $r_0^2 = R_0^2 + \rho_0^2 + 2HR_0\rho_0$.

or $\Pi r_0 = \sqrt{1 + \Pi^2\rho_0^2 + 2H\Pi\rho_0}$.

The following example for $v^2 = Fr_0^2$ is given. Much labour may be saved by using printed forms.

* Instead of $-9{\cdot}677$, so that $L^2 + M^2 + N^2$, to three decimal places, may equal 1·000.

[Example.]

α Lyræ. M = 0·2

R.A. = 18ʰ 31ᵐ 51ˢ; P.M. in R.A. = +0ˢ·0173
N.P.D. = 51° 21′; P.M. in N.P.D. = −0″·295 } Auwers

$\theta = 51^\circ\ 21'$; $\delta\theta = -0{\cdot}295$
$\phi = 277\ 58$; $\delta\phi = +0{\cdot}260$

$\log \sin \theta = +9{\cdot}893$ $\quad +9{\cdot}893$
$\log \cos \phi = +9{\cdot}142$; $\log \sin \phi = -9{\cdot}996$
$\log l = +9{\cdot}035$; $\quad \log m = -9{\cdot}889$; $\log n = +9{\cdot}796$

$\log n = +9{\cdot}796$
$\log \delta\theta = -9{\cdot}470$
$-9{\cdot}266$
$\log \cos \phi = +9{\cdot}142$
$-8{\cdot}408 = -{\cdot}026$

$-9{\cdot}266$
$\log \sin \phi = -9{\cdot}996$
$+9{\cdot}262 = +{\cdot}183$

$\log m = -9{\cdot}889$
$\log \delta\phi = +9{\cdot}415$
$-9{\cdot}304^* = +{\cdot}201$
$\delta l = +{\cdot}175$

$\log l = +9{\cdot}035$
$+9{\cdot}415$
$+8{\cdot}450 = +{\cdot}028$
$\delta m = +{\cdot}211$

$\log \delta l = +9{\cdot}243$; $\quad \delta l^2 = {\cdot}030\ 6$
$\log \delta m = +9{\cdot}324$; $\quad \delta m^2 = {\cdot}044\ 5$
$\log \delta n = +9{\cdot}363$; $\quad \delta n^2 = {\cdot}053\ 2$
$c^2 = {\cdot}128$

$\log \lambda = -9{\cdot}166$
$\log l = +9{\cdot}035$
$-8{\cdot}201 = -{\cdot}016$

$\log \lambda = -9{\cdot}166$
$\log \delta l = +9{\cdot}243$
$-8{\cdot}409 = -{\cdot}026$

$\log \mu = -9{\cdot}914$
$\log m = -9{\cdot}889$
$+9{\cdot}803 = +{\cdot}635$

$\log \mu = -9{\cdot}914$
$\log \delta m = +9{\cdot}324$
$-9{\cdot}238 = -{\cdot}173$

$\log \nu = +9{\cdot}744$
$\log n = +9{\cdot}796$
$+9{\cdot}540 = +{\cdot}347$
$a = +{\cdot}966$

$\log \nu = +9{\cdot}744$
$\log \delta n = +9{\cdot}363$
$+9{\cdot}107 = +{\cdot}128$
$b = -{\cdot}071$

* Change the sign.

$\log L = -9{\cdot}936$
$\log l = +9{\cdot}035$
$-8{\cdot}971 = -{\cdot}094$

$\log M = -9{\cdot}223$
$\log m = -9{\cdot}889$
$+9{\cdot}112 = +{\cdot}130$

$\log N = -9{\cdot}678$
$\log n = +9{\cdot}796$
$-9{\cdot}474 = -{\cdot}298$
$H = -{\cdot}262$

$\log L = -9{\cdot}936$
$\log \delta l = +9{\cdot}243$
$-9{\cdot}179 = -{\cdot}151$

$\log M = -9{\cdot}223$
$\log \delta m = +9{\cdot}324$
$-8\ 547 = -{\cdot}035$

$\log N = -9{\cdot}678$
$\log \delta n = +9{\cdot}363$
$-9{\cdot}041 = -{\cdot}110$
$K = -{\cdot}296$
$a\Pi V = +{\cdot}097$
$K + a\Pi V = -{\cdot}199$
$\log_{\cdot} = -9{\cdot}299$

Assume $\rho_0 = 6{\cdot}4$
$\Pi\rho_0 = {\cdot}128$
$H + \Pi\rho_0 = -{\cdot}134$;

$\log = -9{\cdot}127$
$\log u = +10{\cdot}172$
$\log a = +9{\cdot}985$
$\log au = +10{\cdot}157$

$c^2 = 0{\cdot}128$
$u^2 = 2{\cdot}208$
$-\Pi^2V^2 = -{\cdot}010$
$2{\cdot}326$

$au = +1{\cdot}436$
$-b = +{\cdot}071$
$H\Pi V = -{\cdot}026$
$1{\cdot}481$

And $\frac{1{\cdot}481 \times 10}{2{\cdot}326} = 6{\cdot}4 = \rho_0$

Therefore $\varpi = 0''{\cdot}16$
and $\frac{d\rho}{dt} = -10$

No.	Star.	M.	R.A. 1850.	P.M. in R.A.	N.P.D. 1850.	P.M. in N.P.D.	Observed.		Computed. $v^2=Fr_0^2$		$v^2=Fr_0$		Notes.
							ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	
			h m s	s	° ′	″	″		″		″		
1	α Canis Maj. (*Sirius*)	−1·2	6 38 32	−0·0372	106 31	+1·199	0·27	0?	0·30	0	0·30	0	$\frac{d\rho}{dt}$ var.
2	α Argûs (*Canopus*)	−0·4	6 20 37	·0000	142 37	0 000	0·03	...	...	...	...	...	The P.M.'s are zero.
3	α Boötis (*Arcturus*)	0·0	14 8 49	− ·0799	70 2	+1·977	0·07	− 9	0·38	− 7	0·38	− 7	
4	α Centauri	0·0	14 29 27	− ·4842	150 13	−0·747	0·72	...	large	− 1	large	− 1	$\rho_0=0·6$, $\varpi=1''·67$, in both cases.
5	Aurigæ (*Capella*)	0·1	5 5 37	+ ·0079	44 10	+0·424	0·07	+ 7	0·06	+ 4	0·06	+ 5	
6	β Orionis (*Rigel*)	0·2	5 7 20	− ·0012	98 23	−0·005	0·03	+ 4	0·03	+ 9	0·03	+ 9	
7	α Lyræ (*Vega*)	0·2	18 31 51	+ ·0173	51 21	−0·295	0·12	− 9	0·16	−10	0·16	−10	
8	α Canis Min. (*Procyon*)	0·5	7 31 27	− ·0474	84 24	+1·027	0·23	+ 2	0·14	+ 4	0·14	+ 4	
9	α Orionis	0·9	5 47 3	+ ·0008	82 38	−0·024	0·02	+ 6	0·03	+ 7	0·03	+ 7	
10	α Aquilæ (*Altair*)	1·0	19 43 28	+ ·0351	81 32	−0·384	0·20	− 8	0·32	− 7	0·32	− 7	$\frac{d\rho}{dt}$ var.
11	α Tauri (*Aldebaran*)	1·1	4 27 19	+ ·0035	73 48	+0·184	0·10	+11	0·04	+ 7	0·04	+ 7	
12	α Virginis (*Spica*)	1·1	13 17 18	− ·0044	100 23	+0·018	...	− 4	...	...	0·00	− 2	$\rho_0=265$; $r_0=298$.
13	β Gemin. (*Pollux*)	1·2	7 36 8	− ·0481	61 37	+0·051	0·06	− 7?	...	...	...	...	
14	α Leonis (*Regulus*)	1·3	10 0 23	− ·0182	77 18	−0·018	0·09	+ 1	0·03	+ 5	0·04	+ 4	
15	α Cygni	1·4	20 36 19	− ·0003	45 15	−0·003	0·01	− 7	0·01	− 8	0·01	− 8	
16	α Gemin. (*Castor*)	1·5	7 25 1	− ·0151	57 47	+0·079	...	+ 1?	...	...	...	...	

No.	Star.	M.	R.A. 1850.	P.M. in R.A.	N.P.D. 1850.	P.M. in N.P.D.	Observed.		Computed. $v^2=Fr_o^2$		Computed. $v^2=Fr_o$.		Notes.
							ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	
			h m s	s	° ′	″	″		″		″		
17	γ Orionis	1·8	5 17 5	+ ·002	83 47	+0·04	...	+ 2	0·04	+ 8	0·04	+ 8	
18	ϵ Orionis	1·8	5 28 36	− ·0018	91 18	−0·006	...	+ 6	0·03	+ 9	0·03	+ 9	
19	ζ Orionis	1·8	5 33 12	− ·0008	92 2	−0·010	...	+ 3	0·03	+ 9	0·03	+ 9	
20	ϵ Ursæ Maj.	1·8	12 47 25	+ ·0115	33 14	+0·021	0·08	− 1?	0·06	− 3	0·07	− 3	
21	α Persei	1·9	3 13 38	+ ·0015	40 41	+0·033	0·09	− 6	0·02	+ 1	0·02	+ 2	K zero; computed $\frac{d\rho}{dt}$ doubtful.
22	β Tauri	1·9	5 16 49	+ ·0013	61 31	+0·180	0·06	− 2?	0·04	+ 7	0·04	+ 7	
23	α Ursæ Maj.	1·9	10 54 26	− ·0180	27 26	+0·071	0·05	− 9	0·16	− 6	0·15	− 6	
24	η Ursæ Maj.	1·9	13 41 37	− ·0115	39 56	+0·014	...	− 6	0·01	− 5	0·01	− 5	
25	β Aurigæ	2·0	5 48 32	− ·0065	45 4	+0·011	0·06	− 6	0·01	+ 5	...	...	
26	α Androm.	2·1	0 0 39	+ ·0095	61 44	+0·156	0·06	− 4?	0·03	− 1	0·03	− 1	
27	α Ursæ Min. (*Polaris*)	2·1	1 5 1	+ ·1109	1 29	+0·002	0·05	− 6	0·02	− 6	0·02	− 6	
28	γ Androm.	2·1	1 54 43	+ ·0021	48 24	+0·051	...	− 3	0·02	+ 1	0·00	0	For Fr_o, $\rho_o=300$.
29	α Arietis	2·1	1 58 44	+ ·0127	67 15	+0·134	0·08	− 4	0·03	+ 3	0·03	+ 4	
30	γ Gemin.	2·1	6 29 3	+ ·0023	73 29	+0·035	...	− 4	0·10	+ 6	0·10	+ 6	
31	γ Cassiopeæ	2·2	0 47 42	+ ·0013	30 6	+0·015	0·01	− 4	0·02	− 3	0·01	− 5	

No.	Star.	M.	R.A. 1850.	P.M. in R.A.	N.P.D. 1850.	P.M. in N.P.D.	Observed.		Computed. $v^2=Fr_o^2$		Computed. $v^6=Fr_o$		Notes.
							ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	
			h m s	s	° ′	″	″		″		″		
32	β Androm.	2·2	1 1 21	+ ·0144	55 11	+0·084	0·07	+ 2	0·03	+ 1	0·03	+ 1	
33	δ Orionis	2·2	5 24 21	− ·0014	90 25	+0·005	...	0	0·03	+ 9	0·03	+ 9	
34	γ Leonis	2·2	10 11 42	+ ·0208	69 24	+0 136	...	− 9	...	...	...	...	
35	β Leonis	2·2	11 41 24	− ·0356	74 35	+0 098	0 03	− 3	0·05	+ 1	0·05	+ 1	
36	ζ Ursæ Maj.	2·2	13 17 53	+ ·0134	34 17	+0·022	...	0?	0·07	− 3	0·08	− 3	
37	β Ursæ Min.	2·2	14 51 12	− ·0077	15 14	+0·005	0·02	+ 3	0·03	− 8	0·03	− 8	$\frac{d\rho}{dt}$ var.
38	α Ophiuchi	2·2	17 27 58	+ ·0066	77 20	+0·217	...	0?	0 03	− 5	0·03	− 5	
39	α Coronæ	2·3	15 28 20	+ ·0085	62 47	+0·094	...	+ 7	0·02	− 3	0·04	− 3	
40	γ Cygni	2·3	20 16 51	− ·0001	50 13	−0·020	0·10	− 3	0·02	− 8	0·02	− 8	
41	β Cassiopeæ	2·4	0 1 12	+ ·0658	31 41	+0·190	0·16	0	0 06	− 4	0 06	− 4	
42	α Cassiopeæ	2·4	0 32 2	+ ·0035	34 17	+0 038	0·04	− 3	0·02	− 3	0·02	− 4	
43	β Persei (*Algol*)	2·4	2 58 26	− ·0017	49 38	−0·010	0·05	0	0·01	+ 2	0·01	+ 2	
44	β Ursæ Maj.	2·4	10 52 45	+ ·0085	32 49	−0 047	0·08	+ 1?	0·21	− 1	0·23	− 1	
45	γ Ursæ Maj.	2·4	11 45 55	+ ·0098	35 28	−0·008	0·10	0?	0·09	− 2	0·11	− 2	
46	γ Draconis	2 4	17 53 8	− ·0018	38 29	+0·028	0·07	− 2	0·03	− 9	0·02	− 9	
47	ϵ Pegasi	2·4	21 36 49	+ ·0008	80 49	−0 011	0 08	− 2?	0 02	− 4	...	...	
48	α Pegasi	2 4	22 57 18	+ ·0028	75 36	+0 030	0·08	− 4?	0·02	− 3	...	...	

No.	Star.	M.	R.A. 1850.	P.M. in R.A.	N.P.D. 1850.	P.M. in N.P.D.	Observed.		Computed. $v^2=Fr_0^2$		Computed. $v^2=Fr_0$		Notes.
							ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	
			h m s	s	° ′	″	″		″		″		
49	ϵ Boötis	2·6	14 38 26	− ·0043	62 17	−0·001	...	− 4	...	...	...	...	
50	α Cephei	2 6	21 15 0	+ ·0211	28 3	−0·025	0·06	−14	0·04	− 8	0·03	− 8	
51	β Pegasi	2·6	22 56 30	+ ·0130	62 44	−0·133	...	+ 1	0·45	0	0·08	− 1	
52	β Libræ	2 7	15 8 56	− ·0079	98 50	+0·017	...	− 2	0·00	− 6	0·01	− 4	For Fr_0^2, $\rho_0=375$.
53	α Serpentis	2·7	15 36 53	+ ·0079	83 6	−0·056	...	+ 5	...	...	...	...	
54	ϵ Cygni	2·7	20 40 9	+ ·0280	56 35	−0·335	0·13	...	0·10	− 6	0·10	− 6	
55	δ Leonis	2·8	11 6 7	+ ·0102	68 39	+0 115	...	− 3	0 21	− 1	0 80	0	
56	β Herculis	2 8	16 23 46	− ·0090	68 11	+0·015	...	− 8	0·01	− 9	0 02	− 8	
57	ι Ursæ Maj.	3·2	8 48 55	− ·0441	41 22	+0·247	0·13	...	large	− 2	large	− 2	$\rho_0=0·6$, $\varpi=1''·67$, in both cases.
58	θ Ursæ Maj.	3·2	9 22 48	− ·1041	37 39	+0·564	0 05	...	large	− 3	large	− 3	$\rho_0=0·4$, $\varpi=2''·63$, in both cases.
59	α Herculis	3 2	17 7 49	− ·0019	75 26	−0·030	0·05	...	...	...	...	...	
60	δ Herculis	3·3	17 8 52	− ·005	64 59	+0·15	0·08	...	0 04	−10	0·04	−10	
61	π Herculis	3·4	17 9 50	− ·0035	53 1	−0·005	0·11	...	large	−10	large	−10	$\rho_0=1·0$, $\varpi=1''·00$, in both cases.
62	η Cassiopeæ	3·5	0 40 3	+ ·1346	32 59	+0·481	0·31	...	0·14	− 2	0·13	− 2	
6	η Herculis	3·7	16 37 45	+ ·0028	50 47	+0·077	0·40	...	...	...	...	...	

No.	Star.	M.	R.A. 1850.	P.M. in R.A.	N.P.D. 1850.	P.M. in N.P.D.	Observed.		Computed.				Notes.
									$v^2=Fr_0^2$		$v^2=Fr_0$		
							ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	
			h m s	s	° ′	″	″		″		″		
64	10 Ursæ Maj.	4·2	8 50 53	− ·036	47 38	+0·28	0·20	...	large	+ 1	large	+ 1	$\rho_0=0·21$, $\varpi=4''·76$, in both cases. This star is B A C 3059.
65	70 (*p*) Ophiuchi	4·3	17 57 53	+ ·0131	87 28	+1·109	0·22	...	0·22	− 9	0·22	− 9	
66	θ Cassiopeæ	4·4	1 2 0	+ ·0233	35 39	+0·024	0·23	...	0·05	+ 1	0·04	+ 2	
67	e Eridani	4·4	3 13 55	+ ·270	133 39	−0·75	0·14	...	0·67	+ 9	0·67	+ 9	
68	o^2 Eridani	4·4	4 8 22	− ·1442	97 53	+3·442	0·19	...	large	+ 1	large	+ 1	$\rho_0=0·90$, $\varpi=1''·10$, and $\rho_0=0·93$, $\varpi=1''·08$, respectively
69	δ Ursæ Min.	4·4	18 20 44	+ ·0256	3 24	−0·040	0·03	...	0·03	− 6	0.03	− 6	
70	δ Equulei	4·6	21 7 11	+ ·012	80 36	+0·30	0·04	...	0·09	+ 1	0·08	+ 1	
71	σ Draconis	4·7	19 32 38	+ ·0973	20 36	+1·766	0·25	...	large	0	large	0	$\rho_0=0·23$, $\varpi=4''·35$, and $\rho_0=0·28$, $\varpi=3''·57$, respectively
72	ν^2 Draconis	4·8	17 29 19	+ ·0179	34 43	−0·044	0·28	...	0·11	−10	0·11	−10	
73	ν^1 Draconis	4·9	17 29 14	+ ·0183	34 43	−0·048	0·32	...	0·12	− 9	0·12	− 9	
74	ζ Toucani	5·0	0 12 12	+ ·273	155 45	−1·16	0·06	...	large	+ 7	large	+ 7	$\rho_0=0·39$, $\varpi=2''·54$, in both cases.
75	61 Cygni	5·0	21 0 11	+ ·3444	51 59	−3·230	0·43	...	0·88	− 5	0·87	− 5	
76	μ Cassiopeæ	5·2	0 58 19	+ ·3860	35 49	+1·580	0·18	...	0·40	− 1	0·40	− 1	

No.	Star	M.	R.A. 1850.	P.M. in R.A.	N.P.D. 1850.	P.M. in N.P.D.	Observed.		Computed. $v^2=Fr'^2_\circ$		Computed. $v^2=Fr_\circ$		Notes.
							ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	ϖ	$\frac{d\rho}{dt}$	
			h m s	s	° ′	″	″		″		″		
77	ϵ Indi	5·2	21 51 51	+ ·480	147 24	+ 2·60	0·22	...	0·58	+ 5	0·58	+ 5	
78	Cephei 51 (Hev.)	5·3	6 28 33	− ·0400	2 45	+0·051	0·03	...	0·03	− 6	0·03	− 6	
79	3077 Bradley	5·5	23 6 4	+ ·249	33 40	−0·287	0·18	...	0·25	− 3	0·25	− 3	
80	85 Pegasi	5·8	23 54 21	+ ·0622	63 43	+0 980	0·06	...	0·14	− 1	0·14	− 1	
81	20 Leonis Min.	6·0	9 52 21	− ·0431	57 20	+0·427	0·06	...	0·12	− 8	0·12	− 8	
82	1830 Groombridge	6·4	11 44 19	+ ·3461	51 12	+5·776	0·13	...	large	− 3	large	− 3	$\rho_0=0·18$, $\varpi=5''·56$, in both cases.
83	1618 Groombridge	6·5	10 2 6	− ·1390	39 48	+0 501	0·25	...	large	0	large	0	$\rho_0=0·006$ and 0·005.
84	Σ 1516	7·0	11 5 16	− ·1029	15 43	−0·117	0·15	...	0·21	− 7	0·21	− 7	
85	21185 Lalande	7·5	10 55 8	− ·0453	53 2	+4·716	0·46	...	large	− 4	large	− 4	$\rho_0=0·4$, $\varpi=2''·50$, in both cases.
86	9352 Lacaille	7 5	22 56 0	+ ·5672	126 43	−1·306	0·28	...	large	− 1	large	− 1	$\rho_0=0·115$ in both cases
87	Σ 2398	7·7	18 41 0	− ·1726	30 35	−1·896	0·35	...	...	...	...	...	
88	34 Groombridge	8·1	0 9 52	+ ·2551	46 50	−0·366	0·31	...	0·39	− 1	0·38	− 1	
89	21 258 Lalande	8·5	10 58 0	− ·4004	45 42	−0·943	0 23	...	...	...	...	...	
90	17 415 Oelt. Argel.	9·0	17 37 17	− ·0679	21 33	+1·213	0 20	...	...	...	...	...	
91	11 677 Oelt. Argel.	9·5	11 12 0	−0·507	23 20	−0·21	0 18	...	large	− 7	large	− 7	$\rho_0=0·6$, $\varpi=1''·67$, in both cases.

It thus appears that the results from $v^2 = Fr_0^2$ are identical with those from $v^2 = Fr_0$ in 50 cases; that the results from the former equation are better than those from the latter in 10 cases; but that the differences are important in only 4 cases;* that the results from the latter equation are better than those from the former in 11 cases, but that the difference is important in only 1 case;† and that in the remaining 20 cases the results are either unknown or doubtful.

The present researches therefore show that the results from the two equations are nearly the same, but that those from

$$v^2 = Fr_0^2$$

are slightly preferable to those from the other; and in consequence $v^2 = Fr_0^2$ should certainly be adopted, although no doubt in any special case, as in No. 12, a solution may be obtained from $v^2 = Fr_0$, the circumstance being duly noted.

The results of the above computations may be thus roughly analysed :—

Of the 72 stars observed for parallax, in 49 cases the computations are more or less correct, in 15 cases doubtful, in 1 case wrong, and in 7 cases impossible.

Of the 53 stars observed for radial velocity, in 28 cases the computations are more or less correct, in 13 cases doubtful, in 5 cases wrong, and in 7 cases impossible.

This is a very great improvement upon the revision of 1887, and we are therefore encouraged to look for still closer agreement between observation and theory; at present 3 stars out of every 5 revolve in orbits more or less circular round the common centre, and 2 stars out of every five revolve in orbits more or less elliptic; and it is of course impossible to say at present what degree of circularity will be ultimately obtained.

In our search for further improvement the first matter for consideration is the position of the solar apex, or the point towards which the Sun moves; from the earlier investigations we found

$$\left.\begin{aligned} \alpha &= 259^\circ 8 \\ \beta &= 56 \cdot 4 \end{aligned}\right\} 1850;$$

but recent investigations show that while β is tolerably correct, α should be considerably increased :—

	α	β		α	β
Dunkin	263°7	65°0	L. Struve	273°3	62°7
L. de Ball	269·0	66·8	Plummer	273·1	66·6
Rancken	284·6	58·1	Boss	283·3	45·9
Bischof	285·2	41·5	Stumpe	285·0	53·8
Ubaghs	262·4	63·4	Porter	281·2	49·3

* Nos. 25, 47, 48, and 55. † No. 12.

But if we still assume that the solar orbit is circular, a considerable increase in a will require a considerable increase in the R.A. of the centre ; and this is out of the question in the present state of the theory.

It will therefore be advisable to find corrections to the adopted positions of the Centre and Apex, removing the restriction that they must be 90° apart ; but as this will involve 5 corrections, it will be further advisable to attempt the complete solution and find the 7 corrections, $\Delta\lambda$, $\Delta\mu$, ΔL, ΔM, ΔV, $\Delta\Pi$, and $\Delta\Lambda$, the necessity of which was foreseen in the Revision of 1887, page 539.

We now have

$$F = \Pi^2 V^2 (1 - \Lambda);$$

$$v^2 = F r_0^2;$$

$$r_0^2 = R_0^2 + 2HR_0\rho_0 + \rho_0^2;$$

$$u = \frac{K + a\Pi V + \varpi\Gamma V}{H + \Pi\rho_0};$$

$$\Gamma = L\lambda + M\mu + N\nu;$$

$$\rho_0 = \frac{2V (au - b + H\Pi V)}{c^2 + u^2 - \Pi^2 V^2 + \Lambda\varpi^2 V^2 . \Pi^2 r_0^2};$$

$$\frac{d\rho}{dt} + u\rho_0 = 0;$$

$$\varpi = \frac{1}{\rho_0}.$$

With regard to the factor $\Pi^2 r_0^2$ in the term whose coefficient is Λ, whenever ϖ is equal to or greater than 0″·1, $\Pi^2 r_0^2$ may be taken equal to 1.

Let

$$\epsilon = 2\varpi V (au - b + H\Pi V) - (c^2 + u^2 - \Pi^2 V^2) - \Lambda . \varpi^2 V^2 . \Pi^2 r_0^2,$$

so that if all the quantities have been rightly taken

$$\epsilon = 0.$$

Again let

$$S = \frac{\varpi\left(aV + \frac{d\rho}{dt}\right)}{H + \Pi\rho_0}$$

$$P = \frac{L}{N}; \quad Q = \frac{M}{N};$$

$$p = \frac{\lambda}{\nu}; \quad q = \frac{\mu}{\nu};$$

$$U = \delta l - P\delta n - u (l - nP) + \varpi V (\lambda - \nu P);$$

$$W = \delta m - Q\delta n - u (m - nQ) + \varpi V (\mu - \nu Q);$$

then

$$\frac{d\epsilon}{d\varpi} = 2\mathrm{V}(au - b + \mathrm{H}\Pi\mathrm{V}) - 2\mathrm{S}\left(\frac{d\rho}{dt} \cdot \Pi\rho_0 - \Gamma\mathrm{V}\right) - 2\Lambda\mathrm{V}^2(\varpi + \mathrm{H}\Pi\Big);$$

$$\frac{d\epsilon}{d\lambda} = 2\varpi\mathrm{V}\{u(l - np) - (\delta l - p\delta n)\} + 2\mathrm{SV}\{\Pi(l - np) + \varpi(\mathrm{L} - \mathrm{N}p)\};$$

$$\frac{d\epsilon}{d\mu} = 2\varpi\mathrm{V}\{u(m - nq) - (\delta m - q\delta n)\} + 2\mathrm{SV}\{\Pi(m - nq) + \varpi(\mathrm{M} - \mathrm{N}q)\};$$

$$\frac{d\epsilon}{d\mathrm{L}} = 2\mathrm{SU} + 2\varpi\mathrm{V} \cdot \Pi\mathrm{V}(l - n\mathrm{P})(1 - \Lambda);$$

$$\frac{d\epsilon}{d\mathrm{M}} = 2\mathrm{SW} + 2\varpi\mathrm{V} \cdot \Pi\mathrm{V}(m - n\mathrm{Q})(1 - \Lambda);$$

$$\frac{d\epsilon}{d\mathrm{V}} = 2\varpi(au - b + \mathrm{H}\Pi\mathrm{V}) + 2\mathrm{S}(\Pi a + \varpi\Gamma) + 2\Pi\mathrm{V}(\Pi + \mathrm{H}\varpi) - 2\Lambda\varpi^2\mathrm{V} \cdot \Pi^2 r_0^{\,2};$$

$$\frac{d\epsilon}{d\Lambda} = -\varpi^2\mathrm{V}^2 \cdot \Pi^2 r_0^{\,2};$$

$$\frac{d\epsilon}{d\Pi} = 2\mathrm{S}\left(a\mathrm{V} + \frac{d\rho}{dt}\right) + 2\varpi\mathrm{V}^2(\mathrm{H} + \Pi\rho_0)(1 - \Lambda);$$

where both Γ and Λ are zero at present.

Again let

$$\frac{d\epsilon}{d\lambda} = \mathfrak{A}\frac{d\epsilon}{d\varpi},$$

$$\frac{d\epsilon}{d\mu} = \mathfrak{B}\frac{d\epsilon}{d\varpi},$$

$$\frac{d\epsilon}{d\mathrm{L}} = \mathrm{A}\frac{d\epsilon}{d\varpi},$$

$$\frac{d\epsilon}{d\mathrm{M}} = \mathrm{B}\frac{d\epsilon}{d\varpi},$$

$$\frac{d\epsilon}{d\mathrm{V}} = \mathrm{C}\frac{d\epsilon}{d\varpi},$$

$$\frac{d\epsilon}{d\Lambda} = \mathrm{D}\frac{d\epsilon}{d\varpi},$$

$$\frac{d\epsilon}{d\Pi} = \mathrm{E}\frac{d\epsilon}{d\varpi};$$

then

$$\mathfrak{A} \cdot \Delta\lambda + \mathfrak{B} \cdot \Delta\mu + \mathrm{A} \cdot \Delta\mathrm{L} + \mathrm{B} \cdot \Delta\mathrm{M} + \mathrm{C} \cdot \Delta\mathrm{V} + \mathrm{D} \cdot \Delta\Lambda + \mathrm{E} \cdot \Delta\Pi + \Delta\varpi = 0$$

where $\Delta\lambda$, $\Delta\mu$, $\Delta\mathrm{L}$, &c., are the corrections to be applied to the assumed values of λ, μ, L, &c, and where $\Delta\varpi$ is the observed minus the computed parallax.

Similarly if

$$\mathrm{G} = \frac{\varpi^2(\mathrm{H} + \Pi\rho_0)}{u\mathrm{H}},$$

then

$$\mathfrak{A} \cdot \Delta\lambda + \mathfrak{B} \cdot \Delta\mu + \mathrm{A} \cdot \Delta\mathrm{L} + \mathrm{B} \cdot \Delta\mathrm{M} + \mathrm{C} \cdot \Delta\mathrm{V} + \mathrm{D} \cdot \Delta\Lambda + \mathrm{E} \cdot \Delta\Pi + \mathrm{G} \cdot \Delta\frac{d\rho}{dt} = 0$$

where $\Delta\dfrac{d\rho}{dt}$ is the observed minus the computed radial velocity.

Note I.

The proper motions employed in this article were those found by Dr. Auwers, and were taken from the Greenwich Ten-Year Catalogue of Stars for 1880, excepting:—

No.	Star.	Authority.
2	α Argûs	*Nautical Alm.*, 1896.
4	α Centauri	Elkin, *Mem.*, vol. xlviii. p. 14.
17	γ Orionis	Main, „ „ xix. p. 142.
60	δ Herculis	„ „ „ xxviii. p. 134.
64	10 Ursæ Maj.	B. A. C., 3059.
67	e Eridani	Elkin, *Mem.*, vol. xlviii. p. 175.
70	δ Equulei	Stone, „ „ xxxiii. p. 73.
74	ζ Toucani	Elkin, „ „ xlviii. p. 168.
77	ϵ Indi	„ „ „ xlviii. p. 119.
82	1830 Groomb.	Argelander, Gr.Ten-Year Cat., 1880.
83	1618 Groomb.	„ „ „
84	Σ. 1516	„ „ „
85	21, 185 Lalande	„ „ „
86	9352 Lacaille	Gould, *Mem.*, vol. xlviii. p. 140.
87	Σ. 2398	Lamp., *Astr. Nach.*, 2676 and 2807.
88	34 Groomb.	Argelander, Gr. Ten-Year Cat., 1880.
89	21,258 Lalande	„ „ „
90	17,415 Oeltz. Arg.	„ „ „
91	11,677 Oeltz. Arg.	Geelmuyden, *Astr. Nach.*, 2287.

Note II.

The following parallaxes are employed in the above article, the means of different results being taken when possible. For many of them I am indebted to a Catalogue of Stellar Parallax, by the late Dr. Pritchard (*Ast. Obs. made at the Univ. Obs. Oxford*, No. IV.):—

No.	Star.	Parallax.	Authority.
1	Sirius	0″·23	Henderson, *Mem.* vol. xi. p. 239.
		0·27	Abbe, *M.N.* vol. xxviii. p. 6.
		0·193	Gyldén.
		0·381	Gill and Elkin, *Mem.* vol. xlviii. p. 187.
		0·268	Mean.
2	Canopus	0·028	Elkin, *Mem.* vol. xlviii. p. 183.
3	Arcturus	0·127	Peters.
		0·016	Elkin, *Report Yale Coll. Obs.*
		0·072	Mean.
4	α Centauri	0·913	Henderson and Maclear, *Mem.* vol. xii. p. 370.

No.	Star.	Parallax.	Authority.
		0″·919	Maclear, *Mem.* vol. xx. p. 98.
		0·916	Mean from zenith distances.
		0·880	Moesta at Santiago,
		0·521	„ *Observatory*, vol. iii. p. 662.
		0·512	Elkin, *M.N.* vol. xli. p. 223.
		0·752	Gill and Elkin, *Mem.* vol. xlviii. p. 186.
		0·716	Mean.
5	Capella	0·046	Peters.
		0·095	Elkin, *Yale Coll. Obs.*
		0·070	Mean.
6	Rigel	−0·004	Gill } *Cat. Stellar Par.*
		0·055	Schur }
		0·026	Mean.
7	Vega	0·06 ?	Airy, *Mem.* vol. x. p. 265.
		0·155	W. Struve, Peters, and O. Struve, *M. N.* vol. xiii. p. 74.
		0·103	Peters, *Cosmos*, vol. iii. p. 261.
		0·188	Brünnow, *Dunsink Obs.* vol. ii.
		0·134	A. Hall, *Nature*, vol. xxxv. p. 258.
		0·092	Elkin, *Yale Coll. Obs.*
		0·122	Mean.
8	Procyon	0·123	Auwers.
		0·341	Elkin, *Yale Coll. Obs.*
		0·232	Mean.
9	α Orionis	0·022	Elkin, *Yale Coll. Obs.*
10	Altair	0·181	Struve, *Mem.* vol. xii. p. 29.
		0·214	Elkin, *Yale Coll. Obs.*
		0·198	Mean.
11	Aldebaran	0·102	A. Hall, *Nature*, vol. xxxvi. p. 138.
		0·101	Elkin, *Yale Coll. Obs.*
		0·102	Mean.
13	Pollux	0·057	Elkin, *Yale Coll. Obs.*
14	Regulus	0·089	„ „ „
15	α Cygni	0·012	„ „ „
20	ε Ursæ Maj.	0·081	Pritchard.
21	α Persei	0·087	„
22	β Tauri	0·063	„
23	α Ursæ Maj.	0·046	„

No.	Star.	Parallax.	Authority.
25	β Aurigæ	0″·062	Pritchard.
26	α Androm.	0·058	,,
27	Polaris	0·067	Peters.
		0·025	Schweizer.
		0·015	L. de Ball, *Observatory*, vol. viii. p. 314.
		0·079	Pritchard.
		0·046	Mean.
29	α Arietis	0·080	Pritchard.
31	γ Cassiop.	0·012	,,
32	β Androm.	0·074	,,
35	β Leonis	0·029	,,
37	β Ursæ Min.	0·022	,,
40	γ Cygni	0·102	,,
41	β Cassiop.	0·164	,,
42	α Cassiop.	0·036	,,
43	β Persei	0·058	,,
		0·036	Chase, *Observatory*, vol. xvii. p. 253.
		0·047	Mean.
44	β Ursæ Maj.	0·080	Pritchard.
45	γ Ursæ Maj.	0·099	,,
46	γ Draconis	0·092	Molyneux, *Nature*, vol. xviii. p. 669.
		0·050	Pritchard.
		0·071	Mean.
47	ε Pegasi	0·080	Pritchard.
48	α Pegasi	0·082	,,
50	α Cephei	0·062	,,
54	ε Cygni	0·128	,,
57	ι Ursæ Maj.	0·133	Peters.
58	θ Ursæ Maj.	0·052	Kapteyn, *Cat. Stellar Par.*
59	α Herculis	0·050	Jacob, *Mem.* vol. xxviii. p. 45.
60	δ Herculis	0·084	Leavenworth, *Cat. Stellar Par.*
61	π Herculis	0·11	Belopolsky, ,, ,, ,,
62	η Cassiop.	0·234	Struve, ,, ,, ,,
		0·257	Schweizer, *M.N.* vol. lv. p. 341.
		0·443	Davis, ,, ,, ,,
		0·311	Mean.
63	η Herculis	0·40	Belopolsky, *Cat. Stellar Par.*
64	10 Ursæ Maj.	0·20	,, ,, ,, ,,

No.	Star.	Parallax.	Authority.
65	70 (p) Ophi.	0″·150	Krüger.
		0·286	Schur, *Observatory*, vol. xvii. p. 252.
		0·218	Mean.
66	θ Cassiop.	0·232	Jacoby, *Annals New York Academy of Sciences*, vol. viii.
67	e Eridani	0·136	Elkin, *Mem.* vol. xlviii. p. 178.
68	o^2 Eridani	0·166	Gill, „ „ „ p. 160.
		0·223	A. Hall, *Observatory*, vol. viii. p. 345.
		0·194	Mean.
69	δ Ursæ Min.	0·034	L. de Ball, *Observatory*, vol. viii. p. 314.
70	δ Equulei	0·035	Leavenworth, *Cat. Stellar Par.*
71	σ Draconis	0·246	Brünnow, *Dunsink Obs.* vol. i.
72	ν^2 Draconis	0·28	Belopolsky, *Cat. Stellar Par.*
73	ν^1 Draconis	0·32	„ „ „ „
74	ζ Toucani	0·057	Elkin, *Mem.* vol. xlviii. p. 171.
75	61 Cygni	0·374	Bessel corrected by Peters.
		0·349	Peters, *Cosmos*, vol. iii. p. 260.
		0·505	O. Struve, *Cat. Stellar Par.*
		0·553	Auwers, *M.N.* vol. xxiii. p. 74.
		0·414	Schweizer, *Cat. Stellar Par.*
		0·416	Socoloff, „ „ „
		0·466	Ball, *M.N.* vol. xl. p. 250 and vol. xli. p. 164.
		0·525	Belopolsky, *Cat. Stellar Par.*
		0·270	A. Hall, *Nature*, vol. xxxv. p. 258.
		0·434	Pritchard.
		0·431	Mean.
76	μ Cassiop.	0·342	O. Struve.
		0·084	Schweizer, *Cat. Stellar Par.*
		0·036	Pritchard.
		0·275	Jacoby, *Annals New York Academy Sc.* vol. viii.
		0·184	Mean.
77	ϵ Indi	0·215	Gill and Elkin, *Mem.* vol. xlviii. p. 187.
78	Cephei 51 (Hev)	0·027	L. de Ball, *Observatory*, vol. viii. p. 314.
79	3077 Bradley	0·205	Backlund, *M.N.* vol. xliii. p. 223.
		0·283	Gyldén, „ „ „
		0·070	Brünnow.
		0·185	Mean.

No.	Star.	Parallax.	Authority.
80	85 Pegasi	0″·058	Brünnow, *Dunsink Obs.* vol. ii.
81	20 Leonis Min.	0·062	Kapteyn, *Cat. Stellar Par.*
82	1830 Groomb.	0·226	Peters, *M.N.* vol. xiv. p. 138.
		0·182	Schluter, *M.N.* vol. xiii. p. 132.
		0·140	Wichmann, *M.N.* vol. xiv. p. 138.
		0·141	Döllen, *Cat. Stellar Par.*
		0·034	O. Struve, *M.N.* vol. x. p. 124.
		0·088	Brünnow, *Dunsink Obs.*
		0·126	Kapteyn, *Cat. Stellar Par.*
		0·134	Mean.
83	1618 Groomb.	0·324	Ball, *M.N.* vol. xlv. p. 254.
		0·176	Kapteyn, *Cat. Stellar Par.*
		0·250	Mean.
84	Σ 1516	0·199	Winnecke and Berberich, *Nature*, vol. xxxv. p. 546.
		0·104	L. de Ball, *Nature*, vol. xxxv. p. 546.
		0·152	Mean.
85	21, 185 Lalande	0·501	Winnecke, *Cat. Stellar Par.*
		0·428	Kapteyn, „ „ „
		0·464	Mean.
86	9352 Lacaille	0·285	Gill, *Mem.* vol. xlviii. p. 154.
87	Σ 2398	0·353	Lamp, *Astr. Nach.* 2676 and 2807.
88	34 Groomb.	0·307	Auwers, *M.N.* vol. xxviii. p. 97.
89	21, 258 Lalande	0·262	„ „ „ xxiii. p. 74.
		0·260	Krüger, „ „ „ p. 172.
		0·168	Kapteyn, *Cat. Stellar Par.*
		0·230	Mean.
90	17, 415 Oeltz. A.	0·247	Krüger, *M.N.* vol. xxiii. p. 172.
		0·151	Schweizer, *Cat. Stellar Par.*
		0·199	Mean.
91	11, 677 Oeltz. A.	0·256	Geelmuyden, *Astr. Nach.* 2287.
		0·100	Franz, *Königsberg Obs.* vol. xxxviii.
		0·178	Mean.

Note III.

In the following table the radial velocities ascribed to Dr. Huggins were taken from the *Monthly Notices*, vol. xxxii., p. 361; those ascribed to Greenwich were taken from the Revision of 1887; and those ascribed to Potsdam from the *Monthly Notices*, vol. lii. p. 541.

In the case of any star the mean of the results found by Dr. Huggins and at Greenwich was taken when possible, and considered to be the result of visual observation, and then the mean of this result and that of photography as found at Potsdam was adopted. In the last column $\frac{d\rho}{dt}$ is expressed as usual in mean radii of the earth's orbit per annum, and it is found by dividing miles per second by 2·9.

No.	Star.	Huggins. Miles per sec.	Greenwich. Miles per sec.	Potsdam. Miles per sec.	Adopted. Miles per sec.	Adopted. $\frac{d\rho}{dt}$.
1	Sirius	+20	0	− 9·8	0·0	0?
3	Arcturus	−55	−46·0	− 4·8	−27·4	− 9
5	Capella	+	+26·0	+15·2	+20·6	+ 7
6	Rigel	+15	+19·1	+10·2	+13·6	+ 4
7	Vega	−49	−36·1	− 9·5	−26·0	− 9
8	Procyon	+	+15·5	− 5·7	+ 4·9	+ 2
9	α Orionis	+22	+26·9	+10·7	+17·6	+ 6
10	Altair	...	−24·9	−22·9	−23·9	− 8
11	Aldebaran	+	+32·0	+30·2	+31·1	+11
12	Spica	+	−11·9	− 9·2	−10·6	− 4
13	Pollux	−49	−31·8	+ 0·7	−19·8	− 7?
14	Regulus	+14	+10·1	− 5·7	+ 3·2	+ 1
15	α Cygni	−39	−34·1	− 5·0	−20·8	− 7
16	Castor	+26	+17·8	−18·4	+ 1·8	+ 1?
17	γ Orionis	...	...	+ 5·7	+ 5·7	+ 2
18	ε Orionis	...	...	+16·5	+16·5	+ 6
19	ζ Orionis	...	...	+ 9·3	+ 9·3	+ 3
20	ε Ursæ Maj.	+19	+11·8	−18·8	− 1·7	− 1?
21	α Persei	...	−27·7	− 6·4	−17·0	− 6
22	β Tauri	...	−14·3	+ 5·0	− 4·6	− 2?
23	α Ursæ Maj.	−48	...	− 7·2	−27·6	− 9
24	η Ursæ Maj.	+	...	−16·3	−16·3	− 6
25	β Aurigæ	...	...	−17·5	−17·5	− 6
26	α Androm.	−	−28·2	+ 2·8	−12·7	− 4?
27	Polaris	...	...	−16·1	−16·1	− 6
28	γ Androm.	...	...	− 8·0	− 8·0	− 3
29	α Arietis	...	−14·2	− 9·2	−11·7	− 4
30	γ Gemin.	...	...	−10·3	−10·3	− 4
31	γ Cassiop.	+	−19·3	− 2·2	−10·8	− 4
32	β Androm.	...	...	+ 7·0	+ 7·0	+ 2
33	δ Orionis	...	...	+ 0·6	+ 0·6	0
34	γ Leonis	−	−26·2	−24·0	−25·1	− 9
35	β Leonis	+	...	− 7·6	− 7·6	− 3

No.	Star.	Huggins. Miles per sec.	Greenwich. Miles per sec.	Potsdam. Miles per sec.	Adopted. Miles per sec.	Adopted. $\frac{d\rho}{dt}$.
36	ζ Ursæ Maj.	...	+19·3	−19·4	0·0	0?
37	β Ursæ Min.	...	...	+ 8·9	+ 8·9	+ 3
38	α Ophiuchi	...	−10·9	+11·9	+ 0·5	0?
39	α Coronæ	+	+20·7	+19·9	+20·3	+ 7
40	γ Cygni	−	−13·5	− 4·0	− 8·8	− 3
41	β Cassiop.	...	− 1·2	+ 3·2	+ 1·0	0
42	α Cassiop.	...	...	− 9·5	− 9·5	− 3
43	β Persei	...	...	− 1·0	− 1·0	0
44	β Ursæ Maj.	+	+22·9	−18·2	+ 2·4	+ 1?
45	γ Ursæ Maj.	+	+17·5	−16·5	+ 0·5	0?
46	γ Draconis	...	− 7·2	...	− 7·2	− 2
47	ε Pegasi	...	−14·3	+ 5·0	− 4·6	− 2?
48	α Pegasi	−	−25·5	+ 0·8	−12·4	− 4?
49	ε Boötis	−	...	−10·1	−10·1	− 4
50	α Cephei	...	−41·8	...	−41·8	−14
51	β Pegasi	...	...	+ 4·1	+ 4·1	+ 1
52	β Libræ	...	...	− 6·0	− 6·0	− 2
53	α Serpentis	...	...	+14·0	+14·0	+ 5
55	δ Leonis	+	...	− 8·9	− 8·9	− 3
56	β Herculis	...	...	−22·0	−22·0	− 8

Jamaica:
1896 *September.*

On the curve $y = \left\{\frac{1}{x^2 + \sin^2\psi}\right\}^{\frac{3}{2}}$, *and its connection with an Astronomical Problem.* By Mrs. W. H. Young (Miss Grace Chisholm).

(*Communicated by Professor G. H. Darwin.*)

Introduction. The Problem and its Origin.

The problem of the determination of the orbit of a comet (or other heavenly body) from three observations leads to the study of the following equation :—

$$m - z = \frac{m}{(1 - 2z\cos\psi + z^2)^{\frac{3}{2}}} = \frac{m}{\lambda^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Here m and ψ are certain constants calculated from the observations ; ψ, being the apparent angular distance between the comet and the Sun, lies between 0° and 180° ; m may have any value, but the sign of m is positive or negative according as the apparent path of the comet is convex or concave toward the Sun.*

The unknown quantity z is the ratio of the geocentric distances of the comet and Sun at the time of the mean observation. Hence we perceive that we wish to determine for z a real positive value. The quantity λ, which is obviously always real, is likewise to be taken positive, for it represents the ratio of the heliocentric distances of the comet and Earth at the time of the mean observation.

If we rationalise the equation by squaring both sides we obtain an equation of the eighth degree in z, which, as such, possesses eight complex solutions, conjugate in pairs. One of these is, however, obviously zero ; hence at least one of the other seven solutions is real. The solution $z = 0$ corresponds to the case where we have been observing, not a heavenly body at all, but a speck in our own atmosphere, or even in the telescope itself ; a mistake which is, theoretically at least, always possible.

We have, then, seven solutions to take into account, of which an odd number ($> 0 \leqq 7$) are real. Possibly no one of these real solutions is positive ; in this case the calculations of m and ψ from the observations, or ultimately the observations themselves,

* $m = \frac{\tau_{,}\tau_{,,,}}{2} \frac{\sin P_{,,}}{\sin p_{,,}} \frac{1}{R_{,,}^3}$,

$P_{,,}$ and $p_{,,}$ being the perpendiculars from the Sun and comet respectively at the time of the mean observation on the great circle through the extreme positions of the comet ; $R_{,,}$ is the distance at the same time of the Earth and Sun ; $\tau_{,}$ is the interval of time between the first and second observations multiplied by Gauss's constant ; and $\tau_{,,,}$ is the same quantity for the second and third observations (Oppolzer, p. 362).

are incorrect. But if one single real root be positive then it gives the complete solution of the astronomical problem, and the orbit can be determined. Finally, if more than one real positive solution exists, the observations are insufficient, and the comet must be observed again to discriminate which of the roots is the proper one to take.

We see that the discussion of the roots of the equation (1) has a special interest for astronomers. Now, since the problem as it stands presents difficulties to purely analytical investigation, it was early suggested that it should be treated by geometry, and various geometrical methods have been employed for this purpose. The complete geometrical discussion has not, however, I believe, been given, and I am not aware that the problem has been discussed afresh since the publication of Oppolzer's *Elemente der Bahnbestimmung* (1882). In this last book the geometry is dismissed very shortly, nor is a complete analytical discussion given; and there is one absolute error as to the limits of solution the detection of which led to the writing of this paper.

The geometrical method which I use differs slightly from Oppolzer's and from that used by any of his predecessors. I have added a note on kindred methods to mine, and the results already obtained by them.

I have only to add that I believe I may offer this as a complete geometrical discussion, and that I have also been enabled in this way to give a simple analytical formula as a criterion for the existence of two roots, corresponding to the old formula for the existence of one root only.

I have added a note on Lambert's curvature theorem and its connection with the geometry.

§ 1. *Notation.*

I shall in this paper use the abbreviations s, c for $\sin\psi$ and $\cos\psi$; s is then always positive, and c is positive or negative according as ψ is less or greater than 90°. We have, however, in all cases

$$s^2+c^2=1, \quad \ldots\ldots\ldots\ldots \quad (2)$$

I then put

$$x=z-c. \quad \ldots\ldots\ldots\ldots \quad (3)$$

and replace the equation (1) by the two equations

$$y=\frac{1}{(x^2+s^2)^{\frac{3}{2}}}=\frac{m}{\lambda^3}, \quad \ldots\ldots\ldots\ldots \quad (4)$$

$$my=m-c-x. \quad \ldots\ldots\ldots\ldots \quad (5)$$

§ 2. *Geometry of the Problem.*

Let us now regard x and y as ordinary Cartesian coordinates, the axis of x being horizontal and that of y vertical. The equation (4) then represents a certain plane curve, the equation (5)

a straight line whose intersections with the curve are the object of investigation.

§ 3. *The Astronomical Point, Critical Point, and Critical Line.*

The point of intersection of the straight line (5) with the curve (4) I call, for obvious reasons, the astronomical point.

The line (5) meets the curve once at the point $(-c, 1)$, which I call the critical point. The vertical line through the critical point I call the critical line : its equation is

$$x+c=0,$$

or

$$z=0.$$

The astronomical point, being one for which z is positive, lies to the right of the critical line. Hence we need only consider such intersections of the line (5) with the curve (4) as lie to the right of the critical point.

From the form of the equation we see that $(-m)$ being the cotangent of the inclination of the line to the horizontal axis, m is the distance from the critical line of the point where the line (5) meets the horizontal axis, and is measured to the right if m be positive, to the left if m be negative. This gives a very simple construction for the line (5) :

Find the critical point C and draw the vertical line through it. Measure a distance, m (to the right if m be positive, to the left if m be negative), along the horizonal axis, and join the point so found to C : this line is the line (5).

§ 4. *The Geometry of the Curve.*

Remembering that λ is to be a positive quantity, we see that the curve

$$y=\frac{1}{\lambda^3}=\frac{1}{(x^2+s^2)^{\frac{3}{2}}}$$

lies entirely above the horizontal axis. It is, in fact, only one branch of the algebraic curve of the eighth degree, whose equation is got by rationalising the above equation : this curve possesses a second branch, the reflexion of the first in the horizontal axis, analytically to be distinguished from the upper branch by the fact that in the lower branch λ is always negative. With this lower branch, however, we are not concerned.

Seeing that

$$\frac{dy}{dx}=-3\lambda^{-5}x, \quad \ldots\ldots\ldots \quad (6)$$

we recognise that the horizontal axis is an asymptote to our curve, and that there is one other horizontal tangent, at the point M in the figure, where the curve meets the axis of symmetry ($x=0$). There are no vertical tangents, the steepest inclination being at the inflexions, determined by the equation

$$0=\frac{d^2y}{dx^2}=3\lambda^{-7}(4x^2-s^2),$$

that is to say,

$$x = \pm \frac{s}{2}, \quad \ldots \ldots \ldots \quad (7)$$

this determines the two inflexions V, W.

Substituting for x in (6), we get the tangent of the angle of inclination of the inflexional tangents given by the expression

$$\frac{3}{2}\left(\frac{4}{5}\right)^{\frac{5}{3}} \frac{1}{s^4}.$$

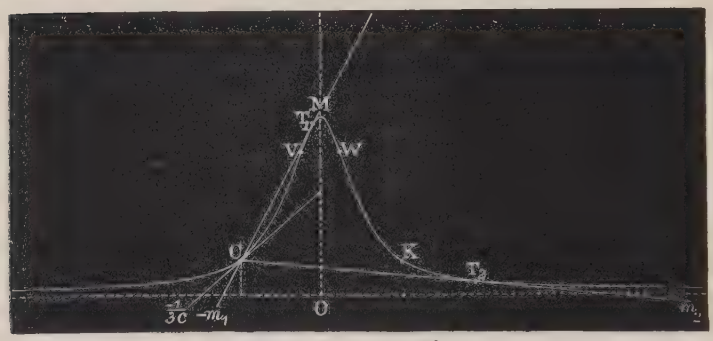


Fig. 1 ($0 < \psi < \tan^{-1} 2$).

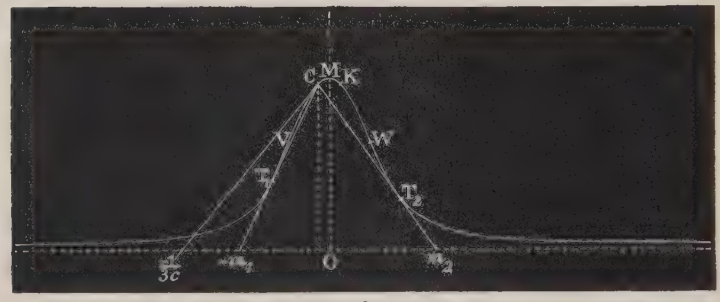


Fig. 2 ($\tan^{-1} 2 < \psi < 90°$).

Thus the nearer ψ is to a right angle, the flatter is the curve (see figures). The form of the curve varies otherwise very little with ψ. The only thing which alters materially with ψ is the position of the critical point with respect to the two inflexions. From 0° to 63° 26′ 8″ ($\tan^{-1} 2$,) the critical point lies to the left of both inflexions, as in fig. 1. When $\psi = 63° 26' 8''$ we have

$$c - \frac{1}{2} s = 0,$$

and the critical point coincides with V. Between this value and 116° 33′ 52″ the critical point lies between the inflexions.

Lastly from 116° 33′ 52″ to 180° the critical point lies to the right of both inflexions.

Now inspection of the figures shows us at once, when the curve has been drawn and the critical point C marked on it, how the number of intersections of the line (5) with the curve which lie to the right of C are determined by the inclination of the line, that is, by the value of m. We see at once that there are three important values of m, corresponding to the tangent *at* the critical point, and the two tangents *from* the critical point, these separate values of m, for which there are respectively no solutions, one solution, two solutions of the problem. The first result got by inspection of our figure is the following: *There cannot be more than two solutions of the astronomical problem.*

For it is obvious that the line can in no case intersect the curve in more than three points, one of which is the critical point itself.

§ 5. *Condition for one Solution only.*

Consider first $0° < \psi < 90°$. The figures then show that when m is negative and sufficiently great, there is a single intersection to the right of C. This goes on until the line touches the curve at the critical point. For this line we have

$$m = -\frac{1}{3c},$$

as may be seen by substituting the coordinates of the critical point in the equation (6).

Hence there is one, and only one, solution if, and only if, m be negative and greater in absolute value than $-\frac{1}{3c}$.

If $90° < \psi < 180°$, the critical point lies to the right of the axis of symmetry, and it is obvious that the same conditions hold if we change the sign of m. Both cases are included if we state the condition for the existence of one, and only one, solution to be

$$1 + 3m\cos\psi < 0.$$

§ 6. *Condition for two Solutions.*—Consider first

$$0° < \psi < \tan^{-1} 2,$$

so that the critical point lies to the left of both inflexions, as in fig. 1. Then we see that from C we can draw two tangents to the curve. The first has its point of contact, T_1, between V and M: this tangent, therefore, makes an acute angle with the horizontal axis and corresponds to a negative value, $-m_1$, of m, which is less in absolute value than $\frac{1}{3c}$. The second point of contact T_2 lies beyond W: this tangent has therefore a positive m, say m_2.

Now inspection shows that from $m = -\frac{1}{3c}$ to $m = -m_1$ the

line (5), falling between the tangents at C and T_1, cuts the curve twice to the right of C, the points of intersection falling one to the right, the other to the left of T_1. Hence for such values of m as lie between $-\frac{1}{3c}$ and $-m_1$ there are two solutions.

From $m=-m_1$ to $m=m_2$ the line falls between the tangents at T_1 and T_2, and does not, therefore, cut the curve at all except at the critical point.

For $m=m_2$ to $m=+\infty$, on the other hand, the line obviously cuts the curve twice to the right of the axis of symmetry, and, therefore, there are again two solutions.

Next let

$$\tan^{-1} 2 < \psi < 90°;$$

the critical point C then lies between V and M, as in fig. 2. We see here that the tangent at T_1 has no longer any interest, since the two intersections lie now to the *left* of C.

Thus from $m=-\frac{1}{3c}$ to $m=m_2$ there are no solutions, and from $m=m_2$ to $m=+\infty$ two solutions.

Next let

$$90° < \psi < \tan^{-1}(-2), \text{ (}i.e.\text{ } 116° 33' 52''\text{)};$$

the critical point C then lies between M and W, but the fig. 2 may otherwise serve our purpose. There will then evidently be no solutions from $m=-\infty$ to $m=m_2$, and two solutions from $m=m_2$ to $m=\frac{1}{-3c}$.

Lastly if

$$\tan^{-1}(-2) < \psi < 180°,$$

the critical point lies to the right of both inflexions, and two solutions are not possible for any value of m.

§ 7. *Analytical Criteria for Existence of two Roots.*

From the equation (6) we see that we have m_1 or m_2 according as we substitute the coordinates of T_1 or T_2 in the expression $\frac{\lambda^5}{3x}$.

To find the coordinates of T_1 or T_2 we write down the equation of the tangent at xy, and make it pass through the critical point $(-c, 1)$.

$$\lambda^5 (y-1) + 3x (x+c) = 0;$$

or, using (4) and substituting for λ^2 its value,

$$\lambda^5 = 4x^2 + 3cx + s^2.$$

Rationalising this we have to determine x the equation

$$(x^2+s^2)^5 = (4x^2+3cx+s^2)^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

One root of this equation is, of course, $x = -c$, corresponding to the tangent at the critical point itself. This root is a double root, and we might, if we chose, divide the whole equation by $(x+c)^2$; this is, however, quite unnecessary, as this root can cause no confusion.

The point of contact T_2 corresponds to the positive root of the equation. When $\psi = 116° 33' 52''$ this root is $\frac{s}{2}$ or $\frac{1}{\sqrt{5}}$ and gradually increases as ψ diminishes, till when $\psi = 0$, the root is very nearly 1·8. It may quite easily be calculated from the equation by Horner's method.

The point of contact T_1 corresponds to a negative root which has only to be calculated when $0° < \psi < 63° 26' 8''$, and which is zero when $\psi = 0$, and gradually increases up to $\frac{1}{\sqrt{5}}$. In calculating this root for small values of ψ we have to be careful, as there is another root of the equation somewhat greater in absolute magnitude than the one we require, which is also zero when $\psi = 0$. This root, however, becomes imaginary for a value of ψ less than $63° 26' 8''$,* so that with care there ought to be no confusion between the two.

In this way a table may be calculated giving for successive values of ψ the limits within which m must lie in order that there may be one, two, or no solutions of the problem. As follows :—

	m			
ψ	One solution.	Two.	None.	Two.
0 to 63° 26′ 8″	$-\infty$ to $-\frac{1}{3c}$	$-\frac{1}{3c}$ to $-m_1$	$-m_1$ to m_2	m_2 to $+\infty$
63° 26′ 8″ to 90°	$-\infty$ to $-\frac{1}{3c}$		$-\frac{1}{3c}$ to m_2	m_2 to $+\infty$
90° to 116° 33′ 52″	$\frac{1}{-3c}$ to ∞		$-\infty$ to m_2	m_2 to $\frac{1}{-3c}$
116° 33′ 52″ to 180°	$\frac{1}{-3c}$ to ∞		$-\infty$ to $\frac{1}{-3c}$	

The actual numbers are printed in Oppolzer's table XIII., b.

§ 8. *Limits of the Solution* $z\ (=x+c)$.

z being the distance of the astronomical point from the critical line, it is at once evident that, when m is positive, z is less than m. For in this case the astronomical point must lie between the critical point and the intersection of the line (5) with the horizontal axis. When, however, m is negative, it is evident that the

* When the tangent at the left-hand lower inflexion passes through the critical point.

astronomical point must lie in that part of the curve which lies between the critical point C and its reflexion K in the axis of symmetry. Hence when m is negative $z < 2$. This inequality is, I believe, new, though it is perfectly obvious from Lambert's curvature theorem without reference to the curve. I give it here chiefly because Oppolzer makes a misstatement on this point when he says that, m being negative, z may have any value from 0 to ∞. This mistake has not, however, crept into his tables.

§ 9. *Note on Lambert's Curvature Theorem.*

Let us imagine the Sun to be at the origin of coordinates O. Now when m is negative, the astronomical point must lie in that part of the curve which lies between the critical point C and its reflexion K in the axis of symmetry, *i.e.* in that part of the curve which is concave toward the Sun, like the apparent path of the comet in this case.

Similarly when m is positive the astronomical point must fall beyond the two points C, K, and therefore in the part of the curve which is convex toward the Sun like the apparent path of the comet in this case. This serves as an easy *memoria technica* for the relation between the sign of m and the concavity or convexity of the apparent path.

Further, since between C and K λ is less than unity, being equal to unity at C and at K, and greater than unity beyond C and K, it follows that when m is positive the astronomical point has its λ greater than unity, and when m is negative less than unity.

Remembering that for the astronomical point λ is the ratio of the heliocentric distances of the comet and Earth, this gives us at once Lambert's curvature theorem, viz. *the comet is nearer the Sun than the Earth is if the apparent path be concave toward the Sun, but is further away than the Earth if it be convex.* The figure is again a great help to the memory.

§ 10. *Note on Geometrical Discussions of the Problem.*

In Gauss's *Theoria Motus*, edited by C. H. Davis, Boston, 1857, references will be found, and abstracts of all the papers on the subject which had appeared up to that date. Of these Encke's original paper on the subject (1848) appears to be the fullest. The curves used by him are, however, complicated, since he treats the equation in the form

$$m \sin 4z = \sin (z - q).$$

Several methods by means of a straight line and curve are given in the book referred to. In all of them, however, the construction of the straight line is made to depend on both

parameters (m, ψ), while the curve is fixed independently. That which has most resemblance to my method is M. Binet's in the *Journal de l'École Polytechnique*, 20 cahier, tome xiii. p. 285. He uses the fixed curve

$$y = \frac{1}{(1+x^2)^{\frac{3}{2}}}.$$

The discussion is, however, very short, and aims only at showing that more than two solutions are impossible, and at obtaining the criterion for a single root. Many years later (1882) Oppolzer used the curve

$$y = \frac{m}{\{1 - 2z\cos\psi + z^2\}^{\frac{3}{2}}}$$

to prove only that more than two solutions are impossible. Here the straight line being inclined at 135° to the axis of z is fixed in direction, and therefore easy to construct, but the discussion is made difficult by the presence of both parameters in the equation to the curve. Oppolzer indeed not only dismisses the geometry very shortly, but gives in his book no general theory, or even an explanation of the mode in which his tables were constructed.

In my own treatment of the subject I have aimed at keeping the two parameters (m, ψ) geometrically distinct, so as to simplify the theory and the classification of the different cases.

Cambridge: March 2, 1897.

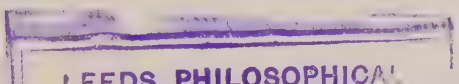
On the Orbit of the November Meteors. By Prof. J. C. Adams.*

It is known to the President and to several members of the Society that I have been for some time past engaged in researches respecting the November Meteors, and allusion is made to some of my earlier results in the last Annual Report. As my investigations are now in some measure complete, and the results which I have obtained appear to me important, I have thought that they may not be without interest for the Society.

In a memoir on the November Star Showers, by Professor H. A. Newton, contained in Nos. 111 and 112 of the *American Journal of Science and Arts*, the author has collected and discussed the original accounts of 13 displays of the above phenomenon in years ranging from A.D. 902 to 1833.

The following table exhibits the dates of these displays, and the Earth's longitude at each date, together with the same particulars for the shower of November last, which have been added for the sake of completeness.

* This paper, by the late Professor Adams, originally appeared in *Monthly Notices*, vol. xxvii. p. 247 (1867 April), but there are now not many copies of the number left in stock. Applications for copies have already been made to the Council, and in view of the return of the *Leonids* in the next few years they have determined to reprint the paper in the current number.—Eds. *M.N.*


LEEDS PHILOSOPHICAL

No.	A.D.	Day and hour. d h	Earth's longitude. ° ′
1	902	Oct. 12 17	24 17
2	931	14 10	25 57
3	934	13 17	25 32
4	1002	14 10	26 45
5	1101	16 17	30 2
6	1202	18 14	32 25
7	1366	22 17	37 48
8	1533	24 14	41 12
9	1602	27 10 O.S.	44 19
10	1698	Nov. 8 17 N.S.	47 21
11	1799	11 21	50 2
12	1832	12 16	50 49
13	1833	12 22	50 49
14	1866	13 13	51 28

From these data Professor Newton infers that these displays recur in cycles of 33·25 years, and that during a period of two or three years at the end of each cycle a meteoric shower may be expected. He concludes that the most natural explanation of these phenomena is, that the November Meteors belong to a system of small bodies describing an elliptic orbit about the Sun, and extending in the form of a stream along an arc of that orbit which is of such a length that the whole stream occupies about one-tenth or one-fifteenth of the periodic time in passing any particular point. He shows that in one year the group must describe either $2 \pm \frac{1}{33 \cdot 25}$, or $1 \pm \frac{1}{33 \cdot 25}$, or $\frac{1}{33 \cdot 25}$ revolutions, or, in other words, that the periodic time must be either 180·0 days, 185·4 days, 354·6 days, 376·6 days, or 33·25 years.

It is seen that the time of the year at which the meteoric shower takes place becomes gradually later and later, and that accordingly the Earth's longitude at that time, or the longitude of the node of the orbit of the meteors, is gradually increasing. Professor Newton finds that the node has a mean motion of 102″·6 annually with respect to the Equinox, or of 52″·4 with respect to the fixed stars; and he remarks that since the periodic time is limited to five possible values, each capable of an accurate determination, and since therefore from the position of the radiant point the other elements of the orbit can be found, it seems possible to compute the secular motion of the node for each periodic time with considerable accuracy; and the actual motion of the node being known, we have thus an apparently simple method of deciding which of the five periods is the correct one.

Soon after the remarkable display of these meteors in November last, I undertook the examination of this question.

From the position of the radiant point as observed by myself, I calculated the elements of the orbit of the meteors, starting with the supposition that the periodic time was 354·6 days, the value which Professor Newton considered to be the most probable one. The orbit which corresponds to this period is very nearly circular, and it readily follows from the ordinary theory that the action of *Venus* would produce an annual increase of about 5″ in the longitude of the node, and that of *Jupiter* an annual increase of about 6″. The calculation of the motion of the node due to the Earth's action presented greater difficulty in consequence of the two orbits nearly intersecting each other. I succeeded, however, in obtaining an approximate solution, applicable to this case, from which it followed that the Earth's action would produce an annual increase of nearly 10″ in the longitude of the node. Thus the three planets above mentioned, which alone, in the case supposed, sensibly affect the motion of the node, would cause a motion of about 21″ annually, or nearly 12′ in 33·25 years. It has been already mentioned that the observed motion of the node is 52″·4 annually, or about 29′ in 33·25 years. Hence the observed motion of the node is totally irreconcilable with the supposition that the periodic time of the meteors about the Sun is 354·6 days. If the periodic time were supposed to be about 377 days, the calculated motion of the node would differ very little from that in the case already considered; while, if the periodic time were a little greater or a little less than half a year, the calculated motion of the node would be still smaller. Hence, of the five possible periods indicated by Professor Newton, four are entirely incompatible with the observed motion of the node; and it only remains to examine whether the fifth period, viz. one of 33·25 years, will give a motion of the node in accordance with observation.

The calculations which have been above described were entirely founded on my own determination of the radiant point. In order to have as secure a basis as possible for the subsequent calculations, I adopted for the position of the radiant point the mean of my own and five other determinations, partly taken from published documents and partly privately communicated to me. These determinations are as follows, the several authorities being placed in alphabetical order: —

	R.A.	Decl.
	° ′	° ′
Adams	148 50	22 10 N.
Baxendell	149 33	22 57
Brünnow	150	22
Challis	149 39	23 12
Herschel	148 9	23 48
Herschel, A.	149	24
Mean	149 12	23 1 N.

Or with reference to the ecliptic,

Long. 143° 22′ Lat. 9° 51′ N.

Starting from this position of the radiant point, and the assumed period, and taking into account the action of the Earth on the meteors as they were approaching it, I obtained the following elliptic elements of their orbit :—

Period	33·25 years (assumed)
Mean distance	10·3402
Excentricity	0·9047
Perihelion distance	0·9855
Inclination	16° 46′
Longitude of Node...	51 28
Distance of Perihelion from Node	6 51

Motion Retrograde

In order to determine the secular motion of the node in this orbit, I employed the method given by Gauss in his beautiful investigation "*Determinatio attractionis &c.*"

It may be proved that if two planets revolve about the Sun in periodic times which are incommensurable with each other, the secular variations which either of these bodies produces in the elements of the orbit of the other would be the same as if the whole mass of the disturbing body had been distributed over its orbit in such a manner that the portion of the mass distributed over any given arc should be always proportional to the time which the body takes to describe that arc. In the memoir just referred to Gauss shows how to determine the attraction of such an elliptic ring on a point in any given position. When this attraction has been calculated for any point in the orbit of the meteors, we can at once deduce the changes which it would produce in the elements of the orbit, while the meteors are describing any given small arc contiguous to the given point. Hence, by dividing the orbit of the meteors into a number of small portions, and summing up the changes corresponding to these portions, we may find the total secular changes of the elements produced in a complete period of the meteors.

In this manner I have found that during a period of 33·25 years, the longitude of the node is increased 20′ by the action of *Jupiter*, nearly 7′ by the action of *Saturn*, and about 1′ by that of *Uranus*. The other planets produce scarcely any sensible effects, so that the entire calculated increase of the longitude of the node in the above-mentioned period is about 28′.

As already stated, the observed increase of longitude in the same time is 29′. This remarkable accordance between the results of theory and observation appears to me to leave no doubt as to the correctness of the period of 33·25 years.

In order to attain a sufficient degree of approximation it is requisite to break up the orbit of the meteors into a considerable number of portions, for each of which the attractions of the elliptic rings corresponding to the several disturbing planets have to be determined; hence the calculations are necessarily very long, although I have devised a modification of Gauss's formulæ which greatly facilitates their application to the present problem. In these numerical calculations I have been greatly aided by my assistants, more especially by Mr. Graham. I am now engaged in obtaining a closer approximation by subdividing certain parts of the orbit of the meteors into still smaller portions, but the results which have been given above cannot be materially changed.

Since I entered upon the foregoing investigation other astronomers have been led, on totally independent grounds, to conclusions which strongly confirm, and are confirmed by, those at which I have myself arrived.

In the *Bullettino Meteorologico dell' Osservatorio del Collegio Romano*, vol. v. Nos. 8, 10, 11, 12, are published four letters from Sig. Schiaparelli, Director of the Observatory of Milan, "*Intorno al corso ed all' origine probabile delle Stelle Meteoriche.*" In these letters the author arrives at the conclusion that the orbits which the meteors describe about the Sun are very elongated, like those of comets, and that probably both these classes of bodies originally come into our system from very distant regions of space. In his last letter, dated 31st December 1866, Sig. Schiaparelli shows that if the August Meteors be supposed to describe a parabola, or a very elongated ellipse, the elements of their orbit, calculated from the observed position of their radiant point, agree very closely with those of the orbit of Comet II. 1862, calculated by Dr Oppolzer. The following table exhibits this agreement :—

	August Meteors.	Comet II. 1862.
Perihelion distance	0·9643	0·9626
Inclination	64° 3′	66° 25′
Longitude of Perihelion	343 28	344 41
Longitude of Node	138 16	137 27
Direction of Motion	Retrograde	Retrograde

Hence it appears probable that the great Comet of 1862 is a part of the same current of matter as that to which the August Meteors belong.

In the letter which has just been referred to Sig. Schiaparelli likewise gives approximate elements of the orbit of the November Meteors, calculated on the supposition that the period is 33·25 years; but as the calculations were founded on an imperfect determination of the radiant point, these elements were not sufficiently accurate, and Sig. Schiaparelli failed to find any

cometary orbit which could be identified with that of the meteors.

Soon after this, on the 21st January 1867, M. Le Verrier communicated to the Academy of Sciences a theory of the origin and nature of shooting stars, very similar in its main features to that of Sig. Schiaparelli, and at the same time gave more accurate elements of the orbit of the November Meteors, his calculations being based on a better determination of the radiant point than that employed by the astronomer of Milan.

In the *Astronomische Nachrichten* of the 29th January Mr. C. F. W. Peters, of Altona, pointed out that the elements given by M. Le Verrier closely agreed with those of Tempel's Comet (I. 1866), calculated by Dr. Oppolzer, and on the 2nd February, Sig. Schiaparelli, having recalculated the elements of the orbit of the meteors on better data than before, himself noticed the same agreement.*

Dr. Oppolzer's elements of Tempel's Comet are as follows :—

Period	33·18 years
Mean distance	10·3248
Excentricity	0·9054
Perihelion distance	0·9765
Inclination	17° 18′
Longitude of Node	51 26
Distance of Perihelion from Node	9 2
Direction of Motion	Retrograde

If these elements be compared with those of the November Meteors which I have given in a former part of this communication, it will be seen that their agreement is remarkably close.

The curious and unexpected resemblance which is thus shown to exist between the orbits of known comets and those of the meteors, both of August and November, opens a wide field for speculation. It is difficult to believe that the coincidences which have been noticed are merely accidental ; but whether or not we are disposed to adopt the ideas of Sig. Schiaparelli as to the intimate relations between meteors and comets, I cannot help thinking that my researches respecting the motion of the node of the November Meteors have settled the question as to the periodic time of these bodies beyond a doubt.

* In the original paper the footnote to this asterisk stands. See ante, p. 246 —Ed., giving the reference to the following note on Schiaparelli's work :—

In the *Ast. Nach.* No. 1629, is contained a paper by Prof. Schiaparelli, "Sur la Relation qui existe entre les Comètes et les Etoiles filantes."

After referring to his letters to Padre Secchi published in the Bulletins of the Observatory of Rome, he reproduces the results as to the August meteor, see *Monthly Notices*, pp. 133–134. He remarks: "In the writings referred to, the position of the Radiant Point of the November Meteors had been taken as the point determined by the American observations of 1833, viz. γ *Leonis*, which is erroneous by several degrees. The new calculation, compared with Dr. Oppolzer's elements of the comet *Ast. Nach.* 1624, the perihelion passage being reduced to Milan Mean Time, is—

	Meteors, 13th Nov.	Comet I, 1866.
Perihelion passage	Nov. 10, 092	Jan. 11, 160
Longitude of Perihelion ...	56° 25′ 9″	60° 28′ 0″
Node	... 231 28 2	231 26 1
Inclination ...	... 17 44 5	17 18 1
Perihelion distance	... 0·9873	0·9705
Excentricity ...	... 0·9046	0·9054
Semi-axis major	... 10·340	10·324
Revolution ...	... 33·250 years	33·176
Motion ...	... retrograde	retrograde

where it is assumed, 1°, the time of max. of the meteors is Nov. 13 $13^h\ 11^m$ Greenwich M.T.; 2°, the position of the radiant point, long. 143° 12′, lat. N. 10° 16′; 3°, period 33·25 years (Newton). The position of the radiant point is that determined by Mr. Herschel, *Monthly Notices*, vol. xxvii. p. 19; but taking the longitude 145° instead of 143° 12′, the discrepancy of 4° in the longitude of perihelion would be made to disappear." M. Schiaparelli asks, Are the meteors to be considered as swarms of minute comets, or as the products of the dissolution of large comets?

Orbit of 44 *Boötis* (H I. 15=Sh 193=Σ 1909).

By S. W. Burnham.

As a double star this was discovered by Herschel August 17, 1781, and the position-angle by a single measure recorded at that time as 29°·9 *nf*, which, according to the mode of reckoning then in use, would give 60°·1. The distance was estimated to be "one-half or three-fourths diameter of S [smaller star]." It does not appear to have been looked at again by Herschel until 1802·25, when, according to his MS. observations, he made the angle 27° *sp* (=243°). These observations are cited in the Catalogue of Herschel II. and South (*Phil. Trans.* 1824), and

also a later note in the MS. of Herschel I., under date of 1803·19, in which it is stated, "the position is not *sp*, as marked in the last observation [1802·25], but *nf*. Distance, one-half diameter of S″. This correction would change the angle of that time from 243° to 63°, and thus apparently show little or no change in the twenty years preceding. From the description the distance would appear to be a little less than at the first observation. The angle was measured by Struve in 1819, and the distance estimated at 1″·5. The next published observation is found in the Catalogue of Herschel and South, who entered it as No. 193 of their list, and measured it on two nights in 1821. They remarked the non-agreement of their angle (229°·1) with the previous measures of Herschel I., and said, "The identity of this star with H I. 15 may be questioned, as it is not impossible that there may be another double star of the first class near the same place with which it has occasionally been confounded. If not, or unless one or both of the stars be variable in magnitude, it is not easy to reconcile the observations." They also compare their position with that of Struve in 1819, and conclude that there can be no rapid motion, as had been suggested by that observer. Struve's observations were resumed in 1826, and show a steady increase in the distance from that time to 1837, with a slight advance in the angle.

Herschel in 1831 (*Mem. R.A.S.*, vol. xlvi.), after giving the measures down to 1830, says :—

"The history of the star 44 *Boötis* is singularly beset with difficulties and apparent contradictions. It is true that an orbit passing nearly through the eye, and so situated that the longer axis of the ellipse into which it is projected shall form an angle of about 6° with the meridian, would account for the remarkable jump the small star seems to have made from one side to the other of the large one ; for its very slow motion between 1781 and 1802 (especially if we admit some conspiring errors in those angles), and for its moderate velocity at present, but a year or two more of observations will probably suffice to render present conjecture needless."

It is probable that the angle of 6° mentioned in this extract is a misprint for 60°, which corresponds to the general apparent movement of the companion. A writer in *Nature* (vol. xiii. 69), in a note on Doberck's orbit, then recently published, suggests that the angle of Herschel in 1802 should not be 63°, as now taken upon the authority of Herschel's MS. correction (making the observation 27° *nf*, instead of 27° *sp* as at first given), but 207° (180°+27°), and points out that with this correction it agrees perfectly with the position of the star in 1802, according to Doberck's orbit. He says :—

"It is now pretty evident from a projection of all the measures to 1875 that the quadrant was correctly registered *sp*, and consequently the quadrant for 1802·24 should be 207°, according to our present mode of reckoning. The angle calculated for this

time from the above orbit is 206° 55′, and this close agreement must be taken as very satisfactory evidence that Dr. Doberck has given us something like the true orbit, notwithstanding the difficulty of the case."

In a subsequent reference to this matter (*Observatory* II. 210), Doberck refers to the suggested correction and its agreement with his orbit, and explains why he rejected in his computation Herschel's position of 63° for 1802, and relied only upon that for 1781.

But these astronomers have overlooked the important fact that under no circumstances could the suggested position-angle be correct. At this time all readings of the micrometer were invariably reckoned from the parallel, that is, added to or subtracted from 270° or 90°, as the case might be. If Herschel inadvertently recorded the angle of the larger instead of the smaller star, or in other words made an error of 180°, and entered it as 27° *sp* instead of *nf*, the true angle would be 243° (270°−27°) instead of 63° (90°−27°), but it could not possibly be 207° (180°+27°) any more than it could be 360°−27°. Such a method of reckoning was never used by Herschel or any one else.

It is possible that Herschel's angles of 1781 and 1802, placing the smaller star in the first quadrant, and in nearly the same direction from the principal star, are substantially correct. There may have been a decided change in the distance during that interval, by a separation and a subsequent approach of the stars, which would not have shown at the time of his observations, even if the distance had been measured. At this time we know absolutely nothing of the form of the apparent orbit, and until this is made to appear with reasonable certainty from the measures which commence with Struve, we cannot safely make any use of the observations prior to that time.

Flammarion in 1878 (*Catalogue des Étoiles Doubles et Multiples*) mentioned the nearly rectilinear motion of the companion, and came to the conclusion that the maximum distance had been reached, and that the companion would then commence to retrace its path towards the primary. He thought that an occultation must have taken place at the beginning of this century, and that this phenomenon would recur on the other side of the orbit about the year 1945 ; and that if the oscillation of the companion with respect to A was symmetrical, the period should be about 280 years.

Two orbits have been computed of this system :

1852	Mädler	181 years	Dorp. Obsns. xiii.
1875	Doberck	261·12 „	A.N. 2064.

The years given above indicate the dates of the last measures used by the respective computers.

G G

It is certainly evident now that there was nothing at these times upon which to base a determination of the periodic time, or the form of the apparent orbit, even if the position of Herschel in 1781 was correct. While the position in the first quadrant might limit the ellipse on the following side, the subsequent measures could not in any manner indicate the extent of the ellipse in the opposite direction, and the period might be anything the computer saw fit to make it. It is hardly necessary to add that the same uncertainty exists at the present time, and so far as one can judge from all the measures, it will be a long time, possibly another hundred years, before a sufficient arc will be passed over to warrant any attempt to investigate the elements of the orbit. The total angular motion during the time covered by the reliable measures is not more than 13°, and the apparent change is nearly rectilinear. There is certainly no doubt that these stars constitute a physical system, and are likely to be hereafter of more than ordinary interest. The components have a considerable common proper motion. The latest determination by Porter gives an annual movement of 0″·401 in the direction of 276°·0.

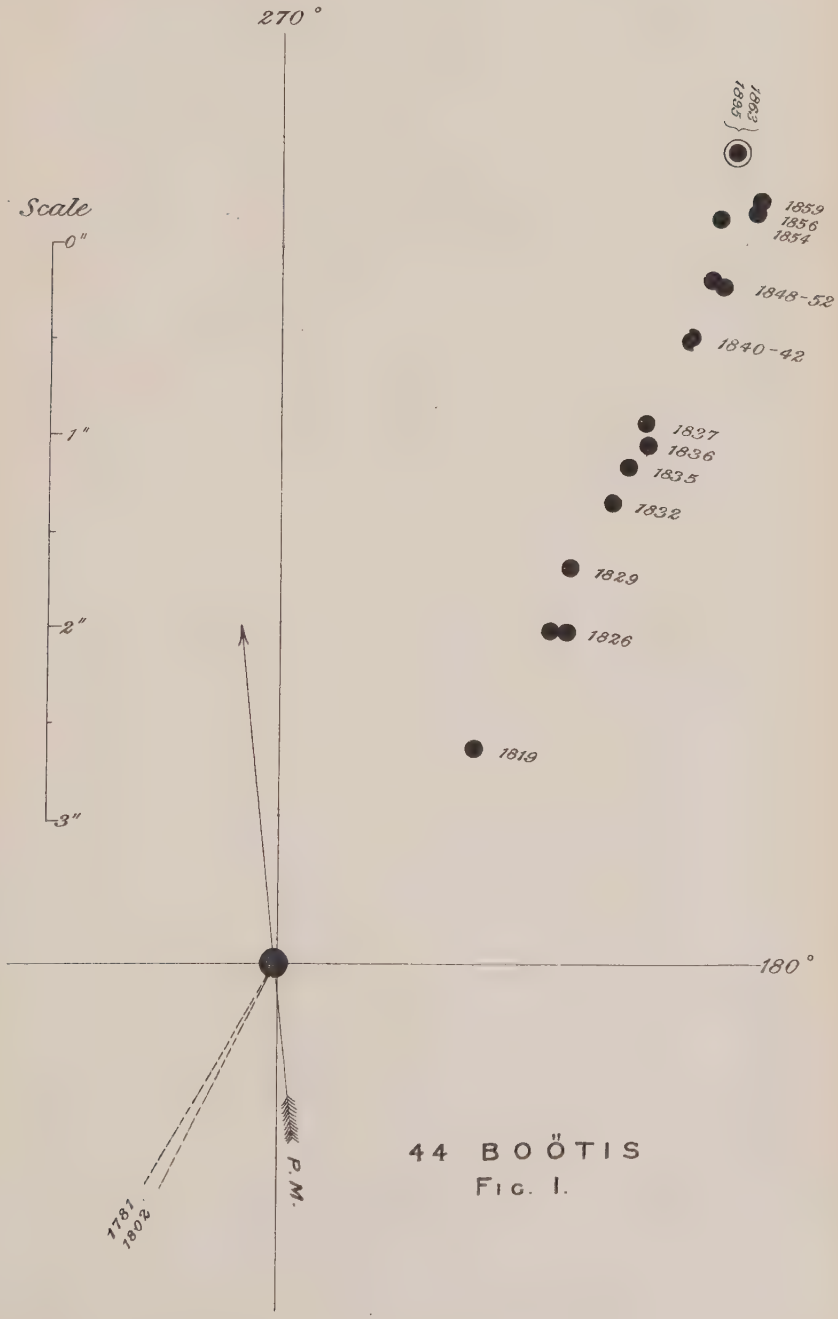
In fig. 1 I have given the early measures by the leading and best observers, laid down to scale in the usual way. These fifteen positions are made up as follows :—

1819	Σ		1848	OΣ and Da	6*n*
1821	Sh	2*n*	1852	Ma	30*n*
1826–1837	Σ (6)	20*n*	1854	Δ	13*n*
1840	OΣ and Da	14*n*	1856	OΣ and Δ	9*n*
1842	OΣ and Da	10*n*	1859	Ma and Δ	10*n*

The last position on the diagram shown by the larger circle is from the mean result of 26 sets of measures from 1863 to 1895. These measures include altogether observations on 108 nights by the following observers :—

O. Struve	12*n*	Hall	32*n*
Dembowski	18*n*	Comstock	5*n*
Schiaparelli	37*n*	Maw	2*n*
	Collins	2*n*	

In fig. 2 all the individual sets of measures above referred to are laid down on square-ruled paper in order to show more clearly the changes in angle and distance at the corresponding dates shown on the lower line of the diagram. It is apparent from inspection that there has been no sensible change in the distance since 1860. There may have been a slight increase in the angle for a short time after this date, but it is by no means certain. We may assume that for the last thirty years these stars have remained relatively fixed so far as the measures are concerned.

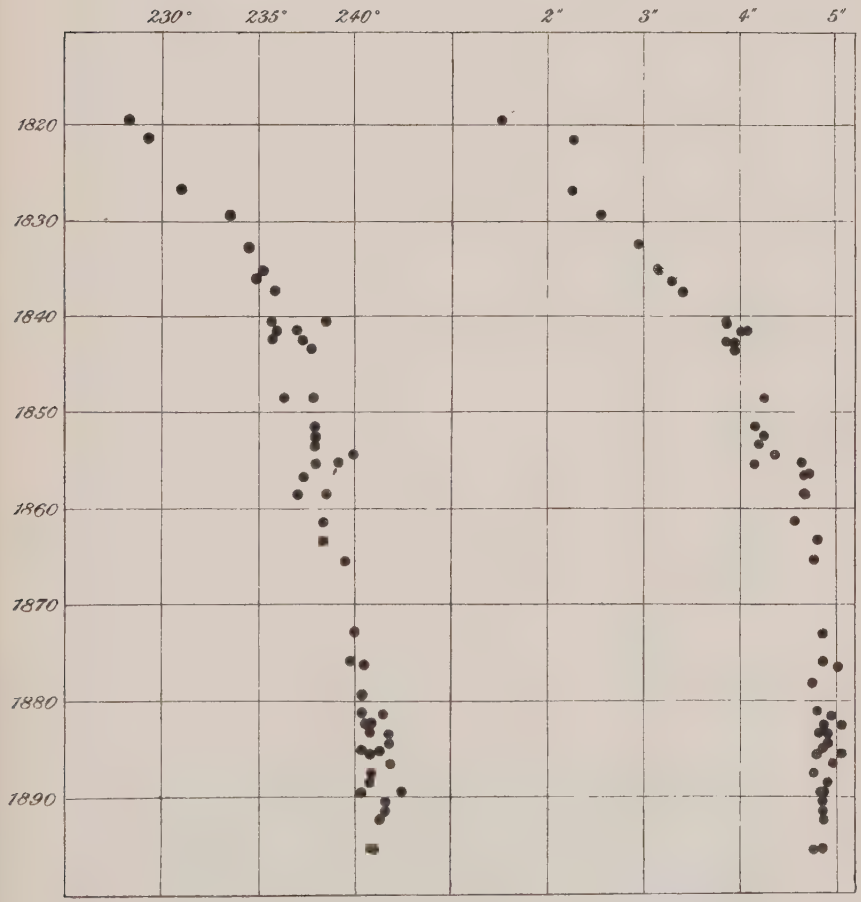


44 BOÖTIS

Fig. 1.

44 BOÖTIS

FIG. 2.



In the following table I have given the variations of the several sets of measures made between 1863 and 1895 from the two means, one of all the measures between these years, and the other of the measures from 1881 to 1895. It will be seen that these variations are practically insensible, and less or no larger than the probable error of any observer when his results are compared one with another.

	1863—1895.		1881—1895.		
	°	″	°	″	
1863·44	−2·5	−0·04	...	...	OΣ
1865·36	−1·3	−0·08	...	...	Δ
1873·00	−0·7	+0·03	...	...	Δ
1876·10	−1·1	+0·02	...	...	Sp.
1876·47	−0·3	+0·19	...	...	Hl.
1879·18	−0·4	−0·08	...	...	Sp.
1881·11	−0·4	−0·04	−0·7	−0·07	Sp.
1881·40	+0·5	+0·10	+0·4	+0·07	Hl.
1882·41	+0·2	+0·21	−0·1	+0·18	Hl.
1882·50	−0·2	+0·04	−0·5	+0·01	Sp.
1883·47	+0·9	+0·08	+0·6	+0·05	Hl.
1883·47	+0·2	−0·01	−0·5	−0·04	Sp.
1884·44	+0·8	+0·09	+0·5	+0·06	Hl.
1884·97	−0·5	+0·04	−0·8	+0·01	OΣ
1885·38	+0·4	+0·20	+0·1	+0·17	Hl.
1885·46	−0·1	−0·03	−0·4	−0·07	Sp.
1886·43	+1·1	+0·12	+0·8	+0·09	Hl.
1887·57	+0·2	−0·08	−0·1	−0·11	Sp.
1888·42	+0·1	+0·07	−0·2	+0·04	Hl.
1889·51	+1·7	−0·04	+1·3	−0·07	Hl.
1889·61	−0·5	+0·01	−0·8	−0·02	Maw
1890·48	+0·8	+0·01	+0·5	−0·02	Hl.
1891·45	+0·7	+0·03	+0·4	0·00	Hl.
1892.58	+0·5	+0·01	+0·2	−0·02	Com.
1895·43	+0·2	+0·03	−0·1	0·00	Col.
1895·49	+0·3	−0·11	0·0	−0·14	Com.

The special object of this paper is not to point out the unwarranted conclusions reached heretofore respecting this star as a binary system, but to call attention to the singular and remarkable arrest of the relative motion of the two stars. For more than thirty years these stars gradually increased their distance from 1″·5 to about 4″·8 at a nearly uniform rate, with a slow advance in the angle ; and then all motion is arrested, and

for a like period of about thirty years, and down to the present time they have remained absolutely at rest so far as one can tell from full and careful sets of measures by the best double star observers. So far as I know, there is no other instance of this kind known in the heavens. There is no lack of examples of double stars where the motion as shown by the various measures is apparently peculiar, and possibly inconsistent with the theory of the mutual attraction of the observed bodies; and various speculations have been advanced to explain the so-called irregular variable motions. A dark or invisible star is usually interposed to account for the observed movements. With all the latitude afforded by the unavoidable errors of observation, which very often appear to assume a systematic form, it is not difficult in any instance to suggest an undiscovered disturbing body which, according to the terms of the proposed hypothesis, shall appear to more or less perfectly account for the observations, but in most instances the disturbing body is more likely to be finally discovered at the smaller end of the telescope than elsewhere.

With respect to this star, I have only to add that it seems to furnish an example, perhaps the only one, of so peculiar motion that the explanation is not readily suggested by the observations themselves. The steady change in the position of the smaller star for about forty years, and the subsequent arrest of its motion, which has continued down to the present time, are both apparently established by unimpeachable observations by the best observers. Of course any attempted explanation at this time would be premature, at the best only speculation, and would have no value, and presumably no interest. The usual dark body will readily suggest itself, and it is easy to imagine one of these stars having an invisible companion, the two moving in a very eccentric orbit, the plane of which is nearly in the line of sight, and to select a period and direction of motion for the hypothetical pair such as will explain approximately not only the motion, but the absence of motion, shown by the observations of the invisible components; and when this is presented with the usual refinements of computation, doubtless for the time being a plausible case could be made out. When the relative motion has been resumed, and a decided change has taken place, there may be no difficulty in determinining with substantial accuracy the general form of the orbit, and explaining the apparent anomalous relative movement of these stars, without assuming the existence of any third member of the system.

On a New Binary of Short Period in the Constellation Dorado $=H_2$ 3683. By T. J. J. See, A.M., Ph.D. (Berlin).

While sweeping over some of the brighter stars in the constellation *Dorado*, January 14, I recognised the duplicity of a yellow star of the 6·5 magnitude; under a power of 520 the components were just well separated, and in brightness rated at

7·8 and 8. This star has proved to be No. 1583 in the Catalogue of Lacaille, by whom it was first observed. In the *Argentine General Catalogue* it is numbered 5325, the companion 5326. The position of the principal star referred to 1900·0 is

$$\alpha = 4^h\ 38^m\ 39^s{\cdot}3\,;\ \delta = -59^\circ\ 8'\ 32''{\cdot}8.$$

While sweeping with the 20-foot reflector at the Cape of Good Hope, 1834 December 3, Sir John Herschel discovered that the star is a fine double. His observations of it are—

(1) *With the 20-foot Reflector.*

t	θ_o	ρ_o	
1834·920	260°·8*	3″ ±	very fine.
1836·030	261·5*	3 ±	very fine, but ill-defined.
1835·475	261·1	3 ±	2*n* Herschel.

(2) *With the 7-inch Equatorial.*

t	θ_o	ρ_o	
1835·925	82°·5	3″·25	
1836·884	80·0	4·20	glimmering, but perfectly steady.
1836·404	81·2	3·72	2*n* Herschel.

The next determinations of the places of the two components are those made at Cordoba by Gould, who gives the following positions :—

		Mag.	Mean Epoch.	α h	m	s	δ °	′	″	No. of Obs.	
5325	Dorado	7	74·82	4	38	14·71	−59	11	24·5	6−6	
5326	Dorado	7¾	73·94	4	38	14·99	−59	11	23·9	1−1	comes sq.
			74·99			15·11			23·5	5−5	

Reduced to polar coordinates these differences in R.A. and Decl. give

t	θ_o	ρ_o	
1874·38	74°·4	2″·23	1*n*
1874·90	71·5	3·16	5*n*

where the mean dates have been used.

If we assign to each result weight proportional to the number of nights we shall obtain

t	θ_o	ρ_o	
1874·82	72°·0	3″·00	6*n* Gould.

While the errors of positions depending on the Meridian Circle are not so small as those of good micrometer measures, there is no doubt that this place fairly represents the position of the companion at the epoch 1874·82.

* These angles should be diminished by 180°.

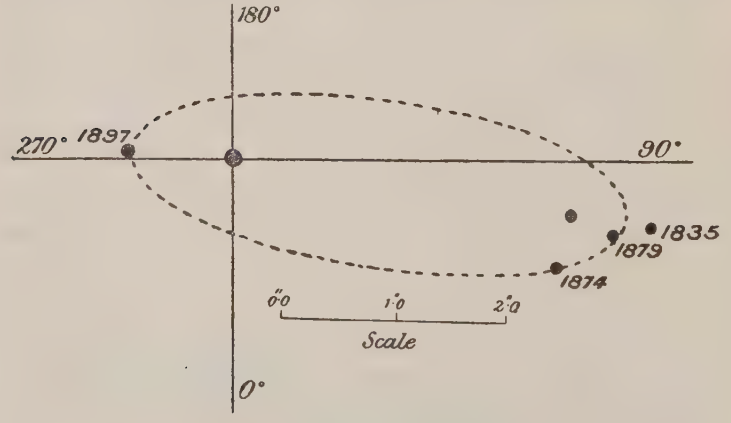
Apparently the only other previous measures of this system are those made by Russell and Hargrave at Sydney in 1879. Their settings of the micrometer gave

t	θ_o	ρ_o	
	°	″	
1879·096	79·1	3·46	1*n* Russell.
1879·096	80·6	3·32	1*n* Hargrave.
1879·096	79·3	3·39	2*n*

Our measures with the 24-inch Clark refractor are as follows :—*

t	θ_o	ρ_o	
	°	″	
1897·036	265·1	0·87	
1897 068	267·0	0·85	
1897·071	263·1	1·03	
1897·058	265·1	0·92	3*n* See.
1897·068	257·7	—	
1897·071	261·3	1·0 ±	
1897·070	259·5	1·0 ±	2–1*n* Cogshall.

On comparing this place with that given by Gould in 1874 we see that a great change has taken place in the last twenty-two years ; indeed, the change since 1879 has been almost as noticeable. It is now clear that when Russell made his measure eighteen years ago the companion was in almost exactly the same place observed by Herschel in 1835, and hence the period is



* Difficult on account of low altitude.

approximately forty-four years. Since it is certain that we may rely on the general accuracy of Gould's place for 1874, it is evident that the motion is direct. From the present position of the companion it is clear that the previous observations relate to the apastron part of the apparent orbit. We may also infer that the eccentricity will lie between 0·6 and 0·7, and that the orbit is considerably inclined about a line of nodes near 85°. The observations do not suffice to fix the other elements, but we may conclude that the apparent orbit will not be very different from that indicated in the figure. The approximate period is obtained by comparing the observations of Herschel and Russell; the constant of areas by comparing the places of Gould and Russell; and hence it is possible to fix the area included between 1879 and 1897. Although the slope of the included arc cannot yet be determined, the approximate path here sketched will be useful to observers.

The rapid motion of this system, which has already carried the companion through one and a half revolutions, combined with the wide angular separation of the components and the high eccentricity of the orbit, renders it an object of extraordinary interest; and hence it merits the most careful attention of observers in the southern hemisphere.

Lowell Observatory, City of Mexico:
1897 January 28.

Estimations of the Magnitude of Nova Aurigæ made at the Radcliffe Observatory, Oxford. By E. J. Stone, M.A., F.R.S., Radcliffe Observer.

In the *Monthly Notices* for 1892 April estimations of the magnitude of *Nova Aurigæ* from 1892 February 3 to March 31 are given. In that interval the star was observed to change from mag. 4·4 to 14·0, the limit of visibility of the Radcliffe instrument. A chart is also given of the comparison stars with their adopted magnitudes.

Subsequent notes in the *Monthly Notices* for 1892 Nov., 1893 Jan., and 1895 Jan. reported that estimations had been made at intervals during 1892, 1893, and 1894; but on the nights, whose dates are there given, the star remained at about 9·7 on the adopted scale.

Since these printed reports examinations have been made on 1895 January 23, 30; September 26; December 7, when no change from 9·7 was detected.

But on 1897 March 10 another examination at once revealed a considerable diminution in brightness; and comparisons made independently by two observers, Mr. Robinson and Mr. McClellan, give the magnitude of the *Nova* as 11·3 on the scale previously adopted. Mr. McClellan also remarked that there

were flickering fluctuations in the brightness of the *Nova*, as seen in previous years, which were not noticed in the comparison stars.

On March 12 the magnitude was estimated by Mr. Wickham as 11·6.

Further Proof of the Rotation Period of Venus.

By Percival Lowell.

(*Communicated by the Secretaries.*)

As the importance of the rotation period of the planet *Venus* is fundamental to the physical constitution of that planet's surface, and as the observations here are absolutely conclusive to the effect that the planet rotates once in an orbital revolution, I send the Society a fresh piece of evidence in the matter which to a certain class of minds is likely to have more weight than any number of drawings.

Drawings are not unnaturally open in the minds of the sceptical to a certain doubt. For what one has not seen oneself is always received, if not with disbelief, at least with something bordering on incredulity ; for which reason drawings are always looked at somewhat askance, especially if the markings depicted appear to be of an unexpected sort. Only when everybody can see the markings for himself are the drawings credited, and then so far as the detection of the markings goes the thing is ancient history. No new thing ever *is* but always *has been* seen. For the sight is never recognised generally until its first recognition is a thing of the past.

As prefatory to what I am about to give, I may mention four points about the markings which are of importance.

1. The markings seen here differ in kind from previous ones. They are not shadings more or less definite, but perfectly distinct markings. I have seen them when their contours had the look of a steel engraving.

2. They are *invariably* visible. Nothing but a very unsteady air can obliterate them.

3. They have been seen by everyone at the observatory who has made any systematic observations of the planets, Mr. Drew, Miss Leonard, and the writer, as well as by others and the drawings of all the observers agree as well as, if not better than, do their drawings of *Mars*. Up to February 6 the number of drawings was 227, showing a very remarkable accordance.

4. This observatory was founded for the express purpose of getting the best atmospheric conditions, which is by far the most important factor in planetary observations ; and what I state of the markings is not surmise, but certainty.

Now although the markings were so distinct as to preclude any possibility of illusion, it is not to be expected that they can

seem so to those who have not observed them. It is therefore advisable to offer evidence which excludes illusion on its face. This is the case with the piece I am now to present.

As a globe turns in consequence of a revolution, orbital or axial, it presents a constantly changing face. On this face any markings must therefore be seen in continually changed positions. If the supposed markings be illusory, the observer of such an illusion must continually be determining unconsciously its position afresh. For such a determination to be performed unconsciously requires—

1. That the observer should estimate unconsciously not a direct quantity but a trigonometric function of the same—namely, the sine of a varying angle, the angular position of the point in question from the centre of the disc.

2. That he should do the like for the sine of a second angle, the angular position of the terminator from the same point.

3. That he should unconsciously combine the two determinations.

It is, I think, evident that such a determination and consequent location would be beyond the automatic possibilities of the most mechanically constituted brain. And this to say nothing of the fact that all the estimates are to be made from the centre of the disc only visible in part; an estimate practically impossible, either unconsciously or consciously.

Lastly, even if the unconscious cerebration should succeed it would speedily be betrayed; for, last but not least, any error it might make—and it would be certain to make some error—would be cumulative. It would increase with the time, for the action, being automatic, would lie beyond the pale of correction. It would gain or lose like a clock when kept under unchanging conditions.

Illusion, therefore, would be quite incapable of producing any nice accordance in the successively observed positions of a point upon a revolving globe. Now it is a very remarkable accordance of the sort which constitutes the fresh evidence of the isochronism of the orbital and axial periods. On measuring the position of *Bilit* on all my drawings made from the time of its first recognition to the close of my observations at Flagstaff, previous to moving to Mexico and tabulating the values, a most interesting result appeared. The measurements were made as follows: First, I measured on the original drawings the relative position of *Bilit* with regard to the limb and the terminator. Then I calculated the position of the terminator theoretically, and then corrected for limit of visibility from my measures of the planet's equatorial and polar diameters. This rendered its position independent of eye estimate. From the two resulting angles I deduced the longitude of *Bilit* on the disc. Now if the axial rotation coincided with the orbital revolution the resulting longitude for *Bilit* should be constant. That it was so, and to a surprising degree of accuracy, the subjoined table will show:—

Venus. Bilit.

	Date.				Longitude, Central Meridian.	Cor. Libration.	Longitude of Bilit from Centre of Disc.	Longitude, Bilit.	Means.	
	h	m	h	m	°	°	°	°	°	°
Oct. 1	4	0–	4	5	31·4	(−·7)	+6·4	37·8	38	37
	4	16–	4	21	31·4	...	+4·3	35·7	36	
3	2	27–	2	37	32·1	...	+4·9	37·0	37	35
	2	45–	2	51	32·1	...	+0·2	32·3	32	
8	2	0–	2	2	34·0	(−·6)	+2·0	36·0	36	36
	2	19–	2	21	34·0	...	+2·5	36·5	36	
	4	42			34·0	...	+2·2	36·2	36	
9	1	59–	2	28	34·4	...	−2·1	32·3	32	34
	2	48–	2	56	34·4	...	+1·0	35·4	35	
	4	50–	4	57	34·4	...	−0·6	33·8	34	
	5	3–	5	8	34·4	...	+0·0	34·4	34	
13	2	55–	3	4	35·9	(−·5)	+0·9	36·8	37	37
14	1	59–	2	12	36·2	...	−0·3	35·9	36	36
	3	13–	3	18	36·2	...	−1·1	35·1	35	
	3	50			36·2	...	+0·3	36·5	37	
15	0	5–	0	14	36·5	...	+0·5	37·0	37	36
	1	0			36·5	...	−3·2	33·3	33	
	2	10–	2	21	36·6	...	+1·2	37·8	38	
	3	48–	3	54	36·6	...	−0·7	35·9	36	
	4	57			36·6	...	+1·0	37·6	38	
16	0	15–	0	20	36·9	...	−4·9	32·0	32	34
	5	0–	5	5	37·0	...	−0·8	36·2	36	
17	3	30			37·3	(−·4)	−3·3	34·0	34	34
19	3	25–	3	34	38·0	...	+1·2	39·2	39	39
	5	5			38·0	...	+0·7	38·7	39	
20	1	0			38·3	...	+1·3	39·6	40	40
23	23	36–	23	45	39·4	...	−2·8	36·6	37	37
25	3	45			40·3	(−·3)	−3·2	37·1	37	37
	4	2–	4	11	40·3	...	−2·4	37·9	38	
28	3	45–	4	0	41·4	...	−4·6	36·8	37	37
	5	0			41·4	...	−4·0	37·4	37	
29	2	59–	3	17	41·7	...	−5·2	36·5	36	36
	4	50–	5	0	41·7	...	−6·0	35·7	36	
31	3	12–	3	32	42·5	(−·2)	−6·7	35·8	36	37
	5	3			42·5	...	−4·0	38·5	39	

Date.				Longitude, Central Meridian.	Cor. Libration.	Longitude of Bilit from Centre of Disc.	Longitude, Bilit.	Means.	
	h m	h m		°	°	°	°	°	°
Nov. 2	2 58			43·3	...	−5·4	37·9	38	38
3	4 2–	4 10		43·7	...	−4·1	39·6	40	40
4	3 10–	3 16		44·0	...	−6·2	37·8	38	
	3 26–	3 37		44·0	(−·1)	−6·0	38·0	38	38
	4 58			44·0	...	−5·3	38·7	39	
5	3 32–	3 38		44·4	...	−7·3	37·1	37	37
7	4 40–	4 49		45·1	...	−6·8	38·3	38	38
9	0 30			45·8	...	−9·5	36·3	36	
	0 48–	0 54		45·8	...	−8·9	36·9	37	37
	1 58			45·8	...	−7·5	38·3	38	

Whence m, the arithmetical mean, $= 36°$; r, the probable error of a single observation, $= 1°·3$; r_0, the probable error of the arithmetical mean $= 0°·2$.

The first column gives the date; the second the longitude of central meridian, the centre of the mean illuminated disc being taken as the zero meridian, and the longitude then corrected for libration; the third, in brackets, the amount of such libration; the fourth, the observed longitude of *Bilit* from the centre of the disc; the fifth, the resulting longitude for *Bilit* from the zero meridian; the sixth, the means.

Now as regards the longitude of *Bilit* on the surface of the planet the determination speaks for itself; but as regards its disclosure of the isochronism of the axial and orbital rotations the evidence it gives is even more conclusive than at first appears. For while it would appear that the longitude of the centre of the disc shifted its position during the observations through $14°·4$ ($45°·8 - 31°·4$), this shift was only the shift of phase (less the libration), the shift as regards the Earth. The shift as regards the Sun, or change in heliocentric longitude of the planet, was in the interval not $14°·4$ but $62°$; and it is the latter taken in connection with the probable error of the longitude of *Bilit* that determines the degree of coincidence between the axial and orbital revolutions. As the probable error of the longitude of *Bilit* was $0°·2$, the probable inaccuracy in the coincidence of the axial and orbital revolutions was therefore less than one three-hundredth of the whole. In other words, this is substantially absolute agreement; for the resulting determination of the longitude of *Bilit* is as close as that of the best observed longitudes on *Mars*.

I may add that observations of *Bilit* up to February 6, the last made, are in accordance with it.

Lowell Observatory, Mexico:
1897 *February* 11.

The Nuclei of a Sun-spot. By T. K. Mellor.

On 1897 February 7 our sky here, which had been persistently overclouded for a number of days, became clear again, and on turning my telescope upon the Sun I saw a large spot with two or three smaller ones around it. It had already passed the centre, and was approaching the western limb, from which it was distant about a quarter of the Sun's diameter.

The spot, although large, was very much of the normal type, the umbra being surrounded by a symmetrical penumbra, the breadth of which was approximately that of the diameter of the umbra ; but in addition to its size there were two features about it which attracted my attention, and made me determine to give it a very careful examination. I also made an outline sketch, a copy of which is appended.

It will be seen that at one place a long arm of the umbra projects into the penumbra, and the end of this arm was cut off and separated from the rest by a bright arc or bridge, which was as brilliant as a facula, and was very conspicuous against the darker background of the penumbra.

But what interested me more was the fact that the umbra itself was not of uniform darkness, as upon one side I saw a large patch, which was sensibly lighter in shade than the remainder, and soon afterwards I recognised something of the same kind upon the other side, although this was of smaller extent. In order to observe this better I removed the diagonal prism and dark glass which I was using and replaced them by a Dawes solar eyepiece. With a weak dark glass and a small diaphragm I was able to distinguish quite clearly that in the umbra there were three dark nuclei, marked A in the sketch, which were intensely black. Although the air was exceptionally steady this observation was difficult on account of the tremor caused by the heat and the constant lengthening and shortening of the focus ;

but during the moments of best definition it appeared to me that these three nuclei were approximately circular, although I could not trace any distinct borders to them. They were all surrounded by the lighter background, and at no point did they touch the inner edge of the penumbra, but were always separated from it by an appreciable margin.

At first I supposed that what I saw was simply an effect of irradiation, but I satisfied myself that the phenomena were real.

The telescope is an equatorial refractor, by T. Cooke & Sons, of York, of 6 inches aperture, and the powers employed were from 60 to 200.

The contrast in colour between the black nuclei and the remainder of the umbra was very slight, but still it was distinctly perceptible, and by careful looking could be traced wherever it extended. These nuclei have been seen by former observers, and may possibly exist in every spot if we could but see them ; but how they are caused or whether they are phenomena of growth or of decay must be left to the investigations of future observers.

Discordances of Index Errors of the Madras Mural Circle in the years 1834–1842 *inclusive.* By A. M. W. Downing, M.A., D.Sc., F.R.S.

In the course of the re-reduction of Taylor's Madras Observations, on which I am engaged, it became necessary to decide whether it was possible to make the declinations independent determinations, or whether it was necessary to make them differential with regard to an adopted standard catalogue, as Taylor had done. The materials for the former course are supplied by the observations of the " Reflecting Collimator," or of the nadir, as we should call it, which were commenced on 1834 August 11, and were continued (with some interruptions) from that date down to the end of the series of observations in 1842.

As the observations which were combined to form the Madras General Star Catalogue (which are those with which we are here concerned) extend from 1831 to 1842, it became necessary to ascertain whether the index errors deduced from the nadir bore any fixed or other determinable relation to those deduced from comparing the corrected circle-readings with the declinations of the Standard Catalogue. It would thus be possible to determine whether index errors corresponding to the nadir could be carried back to the commencement of the series of observations, and carried through the gaps in the nadir observations.

The adopted north latitude of the Madras Observatory, which has been applied to the nadir observations to obtain the index errors is $13^\circ\ 4'\ 8''.1$. The substantial accuracy of this value

appears probable from a re-discussion of the direct and reflection observations of stars given on pp. 94, 95 of vol. i. of the Madras Observations. The corrected value of the latitude thus obtained is 13° 4′ 8″·04.

The standard declinations with which the corrected circle-readings are compared, to deduce the index errors from star observations, are those of Auwers' Mean System (*Ast. Nach.* 1536), and are found by applying the corrections given on page 12 of *Publication der Astronomischen Gesellschaft*, xiv. to the declinations of the Fundamental Catalogue published in the same volume. The stars used for index-error purposes are those of the Fundamental Catalogue situated to the south of dec. +55°.

The following table gives the mean correction to the index error from the nadir deduced from the index error from stars observed on the same day for each year of the series :—

Year.	No. of Comparisons.	Mean Correction.
1834	70	−1″·56 (from Aug. 11)
1835	277	−0·67
1836	253	−1·28
1837	225	−1·31
1838	199	−0·46
1839	115	−1·42
1840	107	−0·56
1841	70	−0·53 (to Sept. 25)
1842	127	−0·87 (from May 17)

The mean correction for the entire series is −0″·96, and the total number of comparisons is 1443.

The apparent correction to the adopted latitude is therefore +0″·96.

The next table shows the result of separating the observations in the four quarters of each year in order to see whether there is any marked change in the correction depending on the season of the year.

	1834.	1835.	1836.	1837.	1838.	1839.	1840.	1841.	1842.
	″	″	″	″	″	″	″	″	″
I.	...	−1·32	−1·31	−1·68	−0·89	−1·74	−0·12	−0·74	...
II.	...	−0·82	−1·86	−1·07	+0·05	−1·69	−1·04	−0·84	−2·13
III.	−1·07	+0·21	−1·06	−1·08	−0·02	−1·41	−0·54	−0·04	−0·55
IV.	−1·82	−0·65	−0·80	−1·24	−1·02	−1·00	−0·55	...	−0·58

The mean correction, the discordance of each mean result from the mean of all, and the R.A. at 10 hours M.T. (representing roughly the average time of star observations) for each group are shown in the next table.

	Mean Correction.	Discordance from Mean.	R.A. at 10 hrs.		Mean Correction.	Discordance from Mean.	R.A. at 10 hrs.
	″	″	h		″	″	h
I.	−1·11	−0·14	7	III.	−0·62	+0·35	19
II.	−1·18	−0·21	13	IV.	−0·96	+0·01	1

These annual discordances are of similar amount, though of quite dissimilar phase, to those found by Auwers from a comparison of Taylor's Catalogue (the declinations of which depend fundamentally on those of Pond's Catalogue of 1830) with the Fundamental Catalogue of the *Astronomische Gesellschaft*. The following table is deduced from his results (*Ast. Nach.* 3195):—

R.A.	Correction to Taylor.	R.A.	Correction to Taylor.
h	″	h	″
7	+0·04	19	+0·02
13	+0·23	1	−0·32

In view of the discordances here exhibited, both of a constant and periodic nature, it appears preferable to form the adopted index errors from a comparison of the corrected circle readings with the declinations of the Standard Catalogue rather than to attempt the formation of independent declinations which would involve so many elements of uncertainty. The former is consequently the course that has been adopted.

"Nautical Almanac" Office:
1897 *March* 8.

Cloud Statistics for Stations in India near the Path of the Moon's Shadow on 1898 *January* 21–22. By A. M. W. Downing, M.A., D.Sc., F.R.S.

Thanks to the kindness of Mr. J. Eliot, F.R.S., Meteorological Reporter to the Government of India, the cloud observations which are embodied in this paper were made at various points near the track of the total eclipse of next January, so as to be a guide to intending observers in the choice of stations.

My request for these observations for the latter part of 1894 January did not reach Mr. Eliot sufficiently early to enable him to make as complete returns for that year as he has done for the corresponding periods of 1895, 1896, and 1897. With the very complete organisation at his command, Mr. Eliot has been able (an opportunity of which he has not been slow to avail himself) to render a signal service to the science of astronomy.

The observations communicated to me consist of the hourly observation of the amount of cloud at each hour of the interval specified at the head of each yearly column in the following

tables. As it did not appear that any hourly variation in the amount of cloud could be detected in the observations, the mean of each hourly amount for the day has been taken to represent the daily amount referring in the mean to the approximate time of totality, or 1^h local mean time.

The columns in the tables under each year give, therefore, the mean daily amount of cloud at the station as thus determined for each day of the period over which the observations extend. The scale adopted is 0—10, where 0 represents a perfectly clear sky, and 10 a perfectly clouded one.

The mean daily amount of cloud for each station is also given.

These mean daily amounts of cloud are represented graphically in the diagram at the end of the paper, where the stations are arranged in order of longitude. The diagram shows clearly the increase in cloudiness as we proceed further towards the east. In fact, from the west coast to 76° E. the average amount of cloud is represented by 1 on the adopted scale; whilst from 78° E. to 85° E. the average amount of cloud is represented by 2. At Darjeeling (the most easterly place of observation) the increase is so great (7·7) that it cannot conveniently be represented on the diagram.

Those who wish for a clear sky would do well, therefore, to locate themselves to the west of the range of hills running from Ahmednagar to Hyderabad. The remarkable general absence of cloud during the latter part of 1896 January is noteworthy.

The detailed observations received from India have been handed over to the custody of the Joint Permanent Eclipse Committee, so as to be available for reference.

Ratnagiri.

East long. 73° 19′; North lat. 17° 0′.

	1895. 10^h-4^h.	1896. 10^h-4^h.	1897. 11^h-3^h.		1895. 10^h-4^h.	1896. 10^h-4^h.	1897. 11^h-3^h.
Jan. 12	...	1	0	Jan. 22	0	1	0
13	...	0	0	23	0	0	0
14	...	0	2	24	1	0	0
15	...	0	0	25	2	0	0
16	...	5	0	26	...	1	4
17	0	0	0	27	...	3	6
18	0	2	5	28	...	2	8
19	0	1	0	29	...	0	8
20	0	0	1	30	...	0	0
21	0	0	0	31	...	0	0

Mean daily amount of cloud 1·1.

Vengurla.

East long. 73° 39′; North lat. 15° 52′.

	1894. 11^h-1^h.		1894. 11^h-1^h.		1894. 11^h-1^h.		1894. 11^h-1^h.
Jan. 22	1	Jan. 25	1	Jan. 28	6	Jan. 31	0
23	1	26	0	29	1		
24	3	27	0	30	0		

Mean daily amount of cloud 1·3.

Marmagao.

East long. 73° 48′; North lat. 15° 24′.

	1895. 10^h-4^h.		1895. 10^h-4^h.		1895. 10^h-4^h.		1895. 10^h-4^h.
Jan. 22	0	Jan. 24	0	Jan. 26	1	Jan. 28	1
23	0	25	0	27	1		

Mean daily amount of cloud 0·4

Satara.

East long. 74° 1′; North lat. 17° 41′.

	1896. 10^h-4^h.	1897. 11^h-3^h.		1896. 10^h-4^h.	1897. 11^h-3^h.
Jan. 12	0	1	Jan. 22	0	2
13	0	0	23	0	2
14	0	3	24	0	2
15	1	3	25	0	2
16	2	0	26	0	2
17	0	0	27	0	7
18	0	2	28	0	8
19	1	1	29	0	8
20	0	4	30	0	0
21	0	2	31	0	0

Mean daily amount of cloud 1·3.

Belgaum.

East long. 74° 31′; North lat. 15° 50′.

	1894. 11^h-1^h.		1894. 11^h-1^h.		1894. 11^h-1^h.		1894. 11^h-1^h.
Jan. 21	0	Jan. 24	6	Jan. 27	0	Jan. 30	0
22	0	25	0	28	6	31	0
23	1	26	0	29	0		

Mean daily amount of cloud 1·2.

Sholapur.

East long. 75° 54′; North lat. 17° 39′.

	1895. 10^h-4^h.	1896. 10^h-4^h.	1897. 11^h-3^h.		1895. 10^h-4^h.	1896. 10^h-4^h.	1897. 11^h-3^h
Jan. 12	...	0	0	Jan. 22	0	0	3
13	...	0	0	23	0	0	0
14	...	0	0	24	...	1	0
15	...	0	1	25	...	0	0
16	...	5	0	26	...	0	4
17	0	0	0	27	...	0	3
18	0	0	0	28	...	0	8
19	0	0	2	29	...	0	8
20	1	0	1	30	...	0	0
21	0	0	2	31	...	0	0

Mean daily amount of cloud 0·8.

Wardha.

East long. 78° 36′; North lat. 20° 44′.

	1897. 11^h-3^h.		1897. 11^h-3^h.		1897. 11^h-3^h.		1897. 11^h-3^h.
Jan. 12	2	Jan. 17	0	Jan. 22	1	Jan. 27	3
13	0	18	2	23	3	28	8
14	0	19	5	24	1	29	7
15	1	20	4	25	4	30	0
6	0	21	3	26	8	31	0

Mean daily amount of cloud 2·6.

Nagpur.

East long. 79° 5′; North lat. 21° 8′.

	1894. 11^h-1^h.	1895. 10^h-4^h.	1896. 10^h-4^h.	1897. 10^h-4^h.		1894. 11^h-1^h.	1895. 10^h-4^h.	1896. 10^h-4^h.	1897. 10^h-4^h.
Jan. 12	...	...	0	3	Jan. 22	1	6	0	7
13	...	...	0	1	23	4	0	1	5
14	...	...	0	0	24	0	1	1	3
15	...	...	0	3	25	1	3	0	3
16	...	...	1	0	26	0	5	0	7
17	...	...	0	0	27	0	...	1	7
18	...	...	0	4	28	2	...	1	7
19	...	3	1	7	29	0	...	2	7
20	...	3	0	5	30	3	...	0	0
21	...	5	0	6	31	0	...	0	0

Mean daily amount of cloud 2·0.

Jubbulpur.

East long. 79° 58′; North lat. 23° 9′.

	1895. 10^h-4^h.	1896. 10^h-4^h.	1897. 11^h-3^h.		1895. 10^h-4^h.	1896. 10^h-4^h.	1897. 11^h-3^h.
Jan. 12	...	0	1	Jan. 22	4	0	5
13	...	0	0	23	...	0	8
14	...	0	0	24	...	0	7
15	...	0	4	25	...	0	1
16	7	1	0	26	...	0	3
17	1	0	0	27	...	0	9
18	0	0	3	28	...	1	2
19	2	0	9	29	...	0	5
20	2	0	4	30	...	1	0
21	5	2	9	31	...	0	2

Mean daily amount of cloud 2·1.

Benares.

East long. 83° 0′; North lat. 25° 20′.

	1895. 10^h-4^h.	1896. 10^h-4^h.	1897. 11^h-3^h.		1895. 10^h-4^h.	1896. 10^h-4^h.	1897. 11^h-3^h.
Jan. 12	...	0	1	Jan. 22	6	0	5
13	...	0	0	23	...	0	10
14	...	0	4	24	...	0	6
15	...	0	5	25	...	0	3
16	5	1	0	26	...	0	4
17	3	0	0	27	...	0	7
18	0	0	4	28	...	1	0
19	3	0	10	29	...	0	0
20	2	1	3	30	...	0	0
21	0	7	8	31	...	0	0

Mean daily amount of cloud 2·1.

Gorakpur.

East long. 83° 23′; North lat. 26° 44′.

	1895. 10^h-4^h.		1895. 10^h-4^h.		1895. 10^h-4^h.		1895. 10^h-4^h.
Jan. 17	4	Jan. 19	3	Jan. 21	2	Jan. 23	8
18	0	20	1	22	8		

Mean daily amount of cloud 3·7

Patna.

East long. 85° 11′; North lat. 25° 33′.

	1894. 11^h-1^h.	1896. 10^h-4^h.	1897. 11^h-3^h.		1894. 11^h-1^h.	1896. 10^h-4^h.	1897. 11^h-3^h.
Jan. 12	...	0	0	Jan. 22	8	0	9
13	...	0	0	23	7	0	5
14	...	0	0	24	2	1	5
15	...	0	1	25	1	0	5
16	...	0	0	26	0	0	5
17	...	0	0	27	0	0	9
18	...	0	1	28	0	9	0
19	...	3	10	29	0	3	0
20	...	0	2	30	0	0	0
21	...	3	8	31	0	0	0

Mean daily amount of cloud 2·0.

Hazaribagh.

East long. 85° 20′; North lat. 23° 59′.

	1894. 11^h-1^h.		1894. 11^h-1^h.		1894. 11^h-1^h.		1894. 11^h-1^h.
Jan. 22	9	Jan. 25	0	Jan. 28	0	Jan. 31	0
23	10	26	0	29	0		
24	0	27	0	30	0		

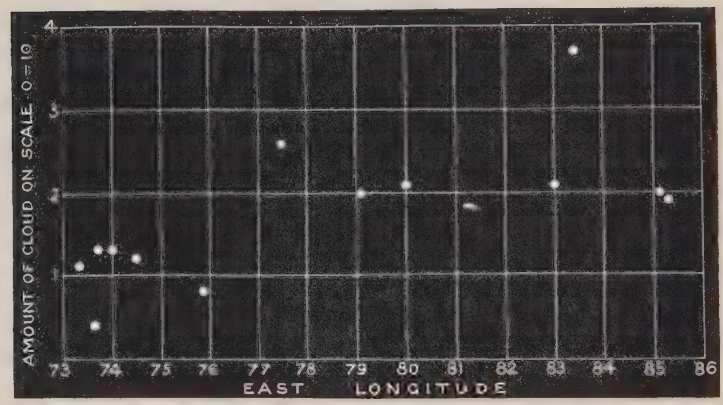
Mean daily amount of cloud 1·9.

Darjeeling.

East long. 88° 18′; North lat. 27° 3′.

	1894. 11^h-1^h.		1894. 11^h-1^h.		1894. 11^h-1^h.		1894. 11^h-1^h.
Jan. 22	5	Jan. 25	3	Jan. 28	9	Jan. 31	10
23	6	26	10	29	10		
24	4	27	10	30	10		

Mean daily amount of cloud 7·7.



Mean amount of cloud in day-time during the latter part of January at stations in India near the path of the moon's shadow on 1898 Jan. 21–22.

Observations taken at Vadsö during the total Eclipse of the Sun, 1896 *August* 9. By Passengers of the S.S. *Neptun* (Norwegian).

(*Drawn up and Communicated by T. C. Porter, Eton College.*)

A. *Direction and Force of Wind.* By E. E. Wilde.

The position taken up was near the edge of the southern cliffs of the island opposite Vadsö, and was about 200 yards E.N.E. of the British Astronomical Association's station; the arrangements, four long pieces of wood laid on the ground pointing N., S., E., and W. respectively, set by the compass *uncorrected for magnetic deviation*. For a flag a very thin piece of old cambric was used, fastened to a crooked stick.

During most of the eclipse there was not sufficient air to lift it, though it flapped gently. At 4.20 there was sufficient air to keep it fully extended, and at 4.25 it was still stronger, calling for the additional note between the 10^m observations arranged to be noted. By 4.50 it had decreased, and continued doing so till totality. The wind observations were taken more than 100 yards distance from Mr. Weir, who was observing temperature, stationed on the rather lower ground to the north-west.

B. *Atmospheric Temperature.* By Thomas Weir (Manchester).

The thermometer used was a mercurial with a long glass spiral in place of the ordinary bulb, specially made by Casartelli & Son (Manchester), very sensitive to change, and ranging from 20° to 120° Fahrenheit. The scale was read with a magnifying glass.

C. *Colour of Sky, Land, and Sea.*

Four observers * noted these during the progress of the eclipse on sheets of paper having the outlines of the scenery filled in before hand, and on each sheet the landscape colour was written on the corresponding part of the already drawn landscape. Two observers took the view east of the Sun, two that to the west, and each entered the time of beginning and ending each sheet as it was used, the position of the Sun being marked by a large dot on each sheet.

	h	m	s		h	m	s
First contact	4	1	6	End of totality	4	58	44
Beginning of totality	4	56	59	Last contact	5	56	42

D. *The Shadow Bands or Striæ.*

Two pairs of observers armed with sticks and sheets watched carefully for these ; but though to almost all the darkness seemed to come on in distinct pulsations, which may be due to the varying rate of the dilatation of the pupil, no shadow bands were observed.

E.

Six other observers, one with a 4-inch refractor, another with two cameras (one of which was provided with a large Nicol), and the remaining four with prepared charts, had intended to map the corona in quadrants, to note its structure, and examine it in two rectangular directions for polarisation, but clouds prevented any work of this kind being done.

Two accounts of the eclipse are appended. The words written at the time by T. C. Porter are copied from his notes almost verbatim ; the other by E. A. Mason, written out just afterwards from a rough account written during the eclipse.

* M. Roberts, J. M. Hughes, G. Hutchings, and John Cordeaux. A brief summary of the observations will be found in the table at the end, arranged chronologically. The time throughout is "Continental."

General Account. By T. C. Porter.

The first contact ($4^h\ 1^m$) was not observed, the sky being overcast with grey stratus cloud, thin overhead, yellowish brown about 45° W. of the Sun, not so deeply coloured the same distance to the E. The point opposite the Sun is yellowish brown. There are a few clear spaces of very light blue sky ($4^h\ 10^m$).

A slight, very small rain falls for a few minutes; patches of sunshine are visible on the hills to the E., which are of a French-blue-mixed-with-white colour. The sky in the W. is gradually clearing, details on the hills to the E. growing clearer ($4^h\ 24^m$).

The light on the horizon to the N.W. grows brighter, but in front, under the Sun (N.E.), it grows rather darker. The clouds near the Sun get darker and more brown, whilst long bluish rifts are drifting slowly from the N.W. towards the N.E. ($4^h\ 31^m$).

At $4^h\ 35^m$ the clouds are distinctly growing to the W. of the Sun. Over it is forming a herring-bone arrangement of cumulo stratus. A slight but very distinct breeze from the N.W. sprung up at $4^h\ 37^m$, the lines of cloud under the Sun are becoming more and more defined and heavier-looking, of a slaty-blue near the horizon.

At $4^h\ 40^m$ seven birds flew past towards the south, all calling.

At $4^h\ 45^m$ four birds flew by towards the north. It has grown sensibly colder; wind very faint from the same quarter. Yellowish ochre gleams on the N.N.E. horizon, then two ravens flying north and croaking loudly went by: they separated, one flying round by N. and W.

The blue sky is getting more like indigo, the clouds the same colour, only darker.

Totality seemed to come on in waves of darkness (*vide* foot-note in table) just *after* the signal given by the British Astronomical Association, the spaces in the sky where the veil of cloud was very thin turning a sickly yellow ochre colour. There is a dead silence, a brilliant red orange sky on the N. and N.W. horizon, *very* dark indigo and muddy French blue clouds, and

Chronological Summary of Observations on the Total Solar Eclipse of 1896 *August* 9.

Time G. Europ. h, m, s	Temp. F. °	Wind Direction	Wind Force	Colour of Land West of Sun / East of Sun
3 30 0	45			
3 45 0	44½	N. slightly E.	*Very* light	
4 0 0	44½	,,	,,	
4 1 6		First contact.		
4 10 0		N. slightly W.	*Very* light	Dist. purple; mid. dist. green from vegetation (A)
4 15 0	45			
4 20 0		N.N.W.	Very light but increasing	
4 25 0	45	N.N.W.	Still increasing	

just under where the Sun lies hidden, and about 2° above the horizon, a few torn, fleecy clouds, bright silver grey, illuminated intensely on the under surfaces (light reflected from the Sun shining on the sea beyond the path of the Moon's umbra). It was quite easy to read the seconds-hand of my watch, and just possible to read the largest letters of the ordinary Wilson S.S. luggage label on my telescope-box cover, standing seven feet away. It was easy to see one's pencil writing, and would have been easy to draw without any artificial light. The sky in one place where it could be seen was a purple colour. The sea between the island and Vadsö was easily seen, but it was impossible to make out objects clearly where they had the turf, &c., of the island we were on for background. The Moon's shadow was seen passing off to the north-east at the close of totality.

General Account. By E. A. Mason.

I was on the island close to Vadsö, away from the rest of my party, undisturbed, and gave my whole senses to observation. The colour of the sky was neutral tint (at the beginning of the eclipse), the sea very grey, and the land a sombre hue. The light began to fade gradually at first, but when totality began a sudden rush of darkness came. A gentleman who was standing some little way off, and whose figure had stood out against the horizon, became enveloped in it and was lost to view. The darkness cleared off as suddenly as it came on. . . . I had near me all the time a reindeer, four sheep, and six ponies. Till the end of totality they were all lying down, then the sheep and ponies got up at once and began to move about, as if they, like myself, felt relieved from the feeling of intense darkness. The reindeer, which was standing grazing when the eclipse began, lay down as it grew darker, as if night had come, and, unlike the other animals, did not move again. The seabirds kept up a perpetual twittering the whole time. The stars were not visible during the eclipse at all. The metal surface of my knife had drops on it after totality, and, though I did not feel it myself, I believe some rain fell in a misty kind of way.

as seen from Vadsö, Varanger Fjord, by some of the passengers of S.S. *Neptun*.

Sea	Sky		Remarks
	West of Sun	East of Sun	
Slate and silver greys (A)	Greys		About 4.15 a slight shower of small rain fell (E)

Time C. Europ. h m s	Temp. F. °	Wind Direction	Force	Colour of Land West of Sun	Colour of Land East of Sun
4 30 0		N.N.W.	Light		
4 35 0	45				Dist. indigo blue; foreground brown and green (C)
4 40 0	44½	N.N.W.	Light		
4 45 0	44				Dist. pale grey blue (B); mid dist. green and grey (B); foreground green (B)
4 48 0					
4 50 0	44	N.N.W.	Light and decreasing	Hills in purple shadow as on a clear night (D)	Dist. blackish blue (C); foreground grey brown (C)
4 51 0					
4 55 0	43½	N.W.	Decreasing		Dist. dull grey blue (B)
4 56 0		N.W.	Still decreasing		Mid dist. dull green (B); foreground dull green (B)
4 56 59	Totality (by calculation)		Stiller, if possible	Dist. hills dark purple (A); foreground yellow green (A)	Dist. hills brown black (C); mid dist. green black (C); foreground lighter green black (C)
4 57 30	Mid totality.				
4 58 44	End of totality (by calculation)				
4 59 0					Dist. hills indigo (C); mid dist. green with grey lights (C); foreground green with golden lights (C)
5 0 0	43½	N.W.	Very light		
5 5 0	44				
5 10 0		N.W.	Very light		
5 12 0					
5 15 0	44½				
5 20 0		N.W.	Very light		
5 30 0	45				
5 35 0		N.W.	Decreased		
5 45 0	45½				
5 56 42		Last contact.			
6 0 0	46	N.	*Very* light, scarcely perceptible		

Observers indicated by the capital letters in brackets:—(A) M. Roberts, observing W. G. Hutchings, observing E. of Sun from the island; (D) J. Cordeaux, observing W. of Sun

N.B.—The events on each horizontal line in this table happened at the time marked in throughout the eclipse.

Sea	Sky: West of Sun	Sky: East of Sun	Remarks
Leaden grey (C)		Grey, blue grey, and high lights pale gold (C)	Clouds began to assume the look of parallelism in W.N.W. to N.N.W. (E)
Dull grey, part greenish yellow (B)		Clouds grey; lights sickly yellow	Much darker. Two ravens crossed the harbour croaking. Oyster catchers on wing calling "peep, peep."
	Clouds dark grey and faint purple (A); "touched along edges with pink" (D)	Heavy looking snow clouds, a strip of blue sky, high lights pale yellow (B)	Some rain (misty) fell
Greyish green (B)			
Dark grey (C); sickly green (B); dark grey under clouds, bright under the horizon gleam (A)	Dark blue grey clouds, bright orange yellow rift near horizon (A)	Black grey clouds, high lights golden, bluish grey near horizon (C)	4.57.—The Moon's shadow visible at a great distance, the darkness coming on in rapid and very distinct pulsations or throbs, about 6 a second (D)
	Some torn feathery clouds bright grey and intensely illuminated on under surfaces about 2° above N.N.E. horizon: they were lower than the other clouds (E)	Near the Sun the horizon was yellowish flame-coloured, fading into pale yellow further east (B)	During totality "nothing invisible far or near" (D) No shadow bands observed either before or after totality
	Grey (A)	Dark blue grey (C)	4.59.—Shadow seen retiring like a thunderstorm gloom (E)
Silver grey (C)			Rapid return of light (D)
			Linear appearance of cloud quite gone (E)

of Sun from the island; (B) J. M. Hughes, observing E. of Sun from the island; (C) from Vadsö Harbour; (E) T. C. Porter, observing from the island.
the first column on the left; thus it is easy to see the progress of the different phenomena

Observations of Comets and Planets made with the 15-inch Dunecht Refractor and Wire Micrometer at the Royal Observatory, Blackford Hill, Edinburgh. By Dr. J. Halm.

(*Communicated by the Astronomer Royal for Scotland.*)

1896.	M.T. Edinburgh. h m s	Δα. m s	Δδ. ′ ″	No. of Comp.	α App. h m s	Log $p\Delta$.	δ App. ° ′ ″	Log $p\Delta$.	Reduction to App. Pl. s		No. of Comp. Star.
				Comet 1896 IV.							
Sept. 14	10 17 5	−1 14·72	− 2 0·7	21, 7	14 44 33·90	9·729	+53 54 18·8	0·741	+0·86	− 2·6	1
				Comet 1897 (*Perrine*, 1896 *Nov.* 2).							
Nov. 4	12 6 1	−1 29·96	− 1 53·9	9, 3	20 18 30·95	9·549	+23 41 23·9	0·850	+2·30	+19.3	2
5	9 22 32	−1 44·36	+ 3 59·4	9, 3	20 17 0·91	9·502	+22 58 14·4	0·764	+2·30	+18·9	3
8	6 10 44	−0 39·51	− 5 16·7	21, 7	20 12 35·39	9·039	+20 42 18·5	0·713	+2·29	+17·6	4
8	6 31 32	−0 16·90	+ 3 42·2	18, 6	20 12 33·81	9·143	+20 41 39·7	0·718	+2·29	+17·5	5
13	7 46 54	−0 40·91	+ 6 52·4	24, 8	20 6 9·29	9·414	+16 57 42·3	0·780	+2·29	+15·4	6
16	8 2 10	+0 37·45	− 4 25·1	24, 8	20 3 1·91	9·455	+14 53 50·0	0·802	+2·29	+14·2	7
16	9 3 9	−1 18·18	− 9 25·4	12, 4	20 2 59·77	9·512	+14 52 17·6	0·823	+2·29	+14·4	8
18	5 36 8	+0 38·93	− 4 14·4	30, 10	20 1 17·80	9·106	+13 39 19·0	0·778	+2·29	+13·5	9
29	5 38 35	+0 35·90	+ 3 17·1	24, 8	19 54 11·32	9·285	+ 7 17 32·7	0·827	+2·34	+ 9·8	10
29	5 59 32	−1 0·49	+ 1 59·0	15, 5	19 54 10	9·336	+ 7 17	0·830	+2·35	+10·0	11
30	5 36 44	−1 12·49	+ 7 0·9	18, 6	19 53 44·24	9·290	+ 6 46 40·0	0·831	+2·36	+ 9·7	12
Dec. 11	6 29 21	−0 19·61	− 5 14·2	24, 8	19 50 20·92	9·461	+ 1 40 49·7	0·858	+2·43	+ 6·9	13
13	5 23 12	+1 17·92	− 3 5·2	24, 8	19 49 59·68	9·378	+ 0 52 31·8	0·860	+2·45	+ 6·4	14
23	5 19 14	+0 43·51	+ 3 26·9	15, 5	19 48 51·10	9·436	− 2 55 29·9	0·871	+2·56	+ 4·6	15
23	5 33 11	−1 16·92	+ 4 19·6	15, 5	19 48 51·20	9·453	− 2 55 37·2	0·870	+2·57	+ 4·9	16

Comet 1896 (*Perrine, Dec.* 8).

1896.	M.T. Edinburgh.	Δα.	Δδ.	No. of Comp.	α App.	Log $p\Delta$.	δ App.	Log $p\Delta$.	Reduction to App. Pl.		No. of Comp. Star.
	h m s	m s	′ ″		h m s		° ′ ″		s	″	
Dec. 11	5 45 4	−2 42·65	+ 3 35·5	18, 6	1 9 32·54	9·223*n*	+ 5 24 14·4	0·836	+4·19	+27·2	17
11	8 41 43	−1 51·45	+ 0 32·7	15, 5	1 10 23·75	8·902	+ 5 21 11·6	0·834	+4·20	+27·2	17
22	7 59 11	+1 11·35	+ 4 14·1	15, 5	2 23 53·69	8·396*n*	+ 1 34 18·5	0·854	+4·50	+23·5	18
23	6 35 14	+1 56·53	+ 7 10·1	18, 6	2 29 38·42	9·159*n*	+ 1 19 53·0	0·857	+4·50	+23·1	19
				◯ 1894 *A.Y.*							
Sept. 15	10 34 10	+1 45·79	+ 5 11·7	15, 5	20 26 46·28	9·183	− 6 45 46·2	0·890	+3·83	+10·9	20
				(106) *Dione.*							
Sept. 17	11 14 35	+1 2·89	−10 56·8	15, 5	23 53 33·67	8·854*n*	− 8 7 45·2	0·898	+4·17	+26·4	21
29	9 57 24	+1 6·39	− 8 18·4	15, 5	23 44 35·76	9·007*n*	− 8 51 20·2	0·900	+4·24	+25·7	22
Oct. 5	10 2 36	+1 2·98	− 4 56·1	15, 5	23 40 28·20	8·713*n*	− 9 6 56·7	0·902	+4·25	+25·3	23
				(92) *Undina.*							
Sept. 17	12 4 3	+0 43·00	+ 3 6·6	15, 5	0 59 24·22	8·977*n*	− 9 37 38·7	0·903	+4·06	+28·9	24
Oct. 12	10 6 22	−1 7·14	− 5 47·2	18, 6	0 41 32·86	8·992*n*	−11 33 41·9	0·908	+4·31	+27·4	25
21	9 53 11	+1 1·92	− 8 45·5	18, 6	0 35 35·33	8·758*n*	−11 51 21·4	0·911	+4·33	+26·3	26
22	11 42 25	−0 39·82	+ 3 22·7	12, 4	0 34 56·34	9·036	−11 52 30·8	0·909	+4·33	+26·2	27
24	10 28 50	−1 47·68	+ 1 58·8	12, 4	0 33 48·48	8·182	−11 53 54·8	0·912	+4·33	+26·1	27

1896.	M.T. Edinburgh.	♂ − ✶ Δα.	Δδ.	No. of Comp.	α App.	Log pΔ.	δ App.	Log pΔ.	Reduction to App. Pl.		No. of Comp. Star.
	h m s	m s	′ ″		h m s		° ′ ″		s	″	
(104) *Klymene.*											
Sept. 29	11 8 58	−1 23·36	−10 28·2	15, 5	0 19 57·92	8·691*n*	− 0 47 33·1	0·867	+4·16	+27·7	28
Oct. 12	9 21 21	+0 46·19	+ 0 30·7	18, 6	0 10 24·28	9·053*n*	− 1 35 47·2	0·870	+4·19	+27·3	29
(225) *Henrietta.*											
Sept. 29	11 58 15	−1 50·42	− 6 5·4	15, 5	23 57 55·15	8·738	+ 9 43 8·1	0·803	+4·07	+27·3	30
Oct. 8	8 8 56	−1 4·61	+ 1 7·2	18, 6	23 53 0·45	9·314*n*	+ 7 57 52·8	0·825	+4·09	+27·7	31
(80) *Sappho.*											
Oct. 8	10 18 18	−0 16·36	+ 5 38·0	18, 6	1 52 54·50	9·304*n*	+16 37 20·6	0·766	+4·31	+27·2	32
21	10 52 14	+0 41·62	− 7 6·5	18, 6	1 43 3·01	8·846*n*	+13 58 12·2	0·772	+4·41	+28·6	33
22	11 17 41	−2 18·31	− 5 14·0	15, 5	1 42 15·95	8·402*n*	+13 45 2·0	0·772	+4·42	+28·6	34
24	10 57 30	−1 38·32	+ 1 2·3	15, 5	1 40 46	8·618*n*	+13 20	0·775	+4·42	+28·7	35
(209) *Dido.*											
Nov. 2	11 1 27	+1 45·70	− 3 43·2	15, 5	2 40 49·36	8·878*n*	+22 52 25·8	0·684	+4·78	+26·6	36
3	11 58 46	+0 51·82	− 6 15·5	15, 5	2 39 55·50	8·336	+22 49 53·5	0·682	+4·80	+26·6	36
4	9 9 35	+0 6·33	− 8 27·3	6. 6	2 39 10·02	9·342*n*	+22 47 41·8	0·717	+4·81	+26·7	36

(195) *Eurykleia.*

1896.	M.T. Edinburgh. h m s	Δα. m s	Δδ. ′ ″	No. of Comp.	α App h m s	Log pΔ.	δ App. ° ′ ″	Log pΔ	Reduction to App. Pl. s ″	No. of Comp. Star.
Nov. 2	11 51 19	−0 58·15	+ 8 0·2	15, 5	2 59 23·78	8·429*n*	+24 0 32·3	0·667	+4·87 +25·0	37
3	12 18 48	−1 56·26	+ 6 20·9	12, 4	2 58 25·68	8·386	+23 58 53·1	0·668	+4·88 +25·1	37
4	9 35 14	−2 47·20	+ 4 49·7	3, 1	2 57 34·75	9·328*n*	+23 57 22·0	0·702	+4·89 +25·2	37
13	9 6 54	−3 39·80	− 5 33·3	12, 4	2 48 58·76	9·282*n*	+23 37 52·3	0·699	+4·96 +26·4	38

(82) *Alkmene.*

Nov. 13	9 53 47	−2 49·61	+ 8 8·2	12, 4	2 33 49·21	8·992*n*	+16 39 29·3	0·751	+4·76 +27·1	39

(133) *Cyrene.*

Dec. 8	12 23 6	+0 12·52	−10 7·8	12, 6	5 53 52·17	8·474*n*	+31 45 58·1	0·555	+6·07 + 6·7	40

(8) *Flora.*

Nov. 29	10 23 38	+2 54·70	− 4 56·9	12, 4	4 43 40·65	9·171*n*	+14 15 49·3	0·776	+5·15 +16·6	41
Dec. 8	11 17 7	−1 59·83	− 1 34·2	15, 5	4 33 38·52	7·695*n*	+14 35 56·8	0·764	+5·26 +16·8	42
11	9 18 21	+2 5·74	− 1 18·9	12, 4	4 30 36·02	9·191*n*	+14 44 23·2	0·772	+5·27 +17·5	43
15	11 48 38	+1 16·79	+ 4 36·3	15, 5	4 26 34·94	8·965	+14 57 46·0	0·764	+5·30 +17·7	44

Mean Places of Comparison Stars.

No.	R.A. 1896·0.	Decl. 1896·0.	Authority.
	h m s	° ′ ″	
1	14 45 47·76	+53 56 22·1	A.G.Z., Cambr.
2	20 19 58·61	+23 42 58·5	A.G.Z., Berlin B.
3	20 18 42·97	+22 53 56·1	,,
4	20 13 12·61	+20 47 17·6	,,
5	20 12 48·42	+20 37 40·0	,,
6	20 6 47·91	+16 50 34·5	A.G.Z., Berlin A.
7	20 2 22·17	+14 58 0·9	½ (Bessel + Lamont).
8	20 4 15·66	+15 1 28·6	A.G.Z., Berlin A.
9	20 0 36·58	+13 43 19·9	Lamont.
10	19 53 33·08	+ 7 14 5·8	½ (Bessel + Lamont).
11	19 55 9	+ 7 15	B.D. + 7° 4330.
12	19 54 54·37	+ 6 39 29·4	Lamont.
13	19 50 38·10	+ 1 45 57·0	A.G.Z., Albany.
14	19 48 39·31	+ 0 55 30·6	Bessel – Weisse.
15	19 48 5·03	− 2 59 1·4	,,
16	19 50 5·55	− 3 0 1·7	München.
17	1 12 11·00	+ 5 20 11·7	½ (Schj. + A.G.Z., Alb.)
18	2 22 37·84	+ 1 29 40·9	A.G.Z., Albany.
19	2 27 37·39	+ 1 12 19·8	,,
20	20 24 56·66	− 6 51 8·8	Lamont.
21	23 52 26·61	− 7 57 14·8	Bessel – Weisse.
22	23 43 25·13	− 8 43 27·5	,,
23	23 39 20·97	− 9 2 25·9	,,
24	0 58 37·16	− 9 41 14·2	Schjell.
25	0 42 35·69	−11 28 22·1	,,
26	0 34 29·08	−11 43 2 2	½ (Armagh + Bruxelles).
27	0 35 31·83	−11 56 19·7	Bessel – Weisse.
28	0 21 17·12	− 0 37 32·6	Göttingen.
29	0 9 33·90	− 1 36 45·2	,,
30	23 59 41·50	+ 9 48 46·2	½ (Bessel + Rümk.).
31	23 54 0·97	+ 7 56 17·9	Bonn Beob.
32	1 53 6·55	+16 31 15·4	A.G.Z., Berlin A.
33	1 42 16·98	+14 4 50·1	Bessel – Weisse.
34	1 44 29·84	+13 49 47·4	Lalande.
35	1 42 20	+13 18	B.D. + 13° 276.
36	2 38 58·88	+22 55 42·4	A.G.Z., Berlin B.
37	3 0 17·06	+23 52 7·1	,,

No.	R.A. 1896·0. h m s	Decl 1896·0. ° ′ ″	Authority.
38	2 52 33·60	+23 42 59·2	A.G.Z., Berlin B.
39	2 36 34·06	+16 30 54·0	A.G.Z., Berlin A.
40	5 53 33·58	+31 55 59·2	½ (Bessel + Bruxelles).
41	4 40 40·80	+14 20 29·6	Bessel − Weisse.
42	4 35 33·09	+14 37 14·2	„
43	4 28 25·01	+14 45 24·6	„
44	4 25 12·85	+14 52 52·0	½ (Arm. + Bruxelles).

Notes.

Sept. 14.—Comet exceedingly faint.

Nov. 4.—(195). Observation interrupted by clouds.

Nov. 4.—Comet exceedingly faint, scarcely visible.

Nov. 13.—During the observation of the comet a strong wind shook the instrument. The comet resembles a faint nebula with a distinct nucleus of about 13th magnitude.

Nov. 16.—Comet very faint; fog and moonlight.

Nov. 18.—Comet well seen, though faint.

Dec. 13.—Comet very faint. Hazy sky and moonlight.

Dec. 22.—Comet faint.

Dec. 23.—*Perrine*, Nov. 2, is a very distinct round nebulosity.

Dec. 23.—*Perrine*, Dec. 8, has an oval form and nucleus of 11th magnitude.

Transit Circle Observations of Comet Swift (1896 *April* 13) *at the Radcliffe Observatory, Oxford.*

(*Communicated by E. J. Stone, Esq., M.A., F.R.S., Radcliffe Observer.*)

The following comet observations, excepting May 2, were made with illuminated wires. A power of 80 was used on each night :—

Greenwich Mean Solar Time of Transit (sub polo).	Observer.	Apparent R.A. of Comet.	Apparent N.P.D. of Comet (Uncorrected for Parallax).	Parallax q.	Log $(q \times \Delta)$.
1896. h m s		h m s	° ′ ″	″	
Apr. 30 12 27 57·6	W.	3 0 40·03	37 55 18·5	13·8	0·9327
May 1 12 18 53·5	W.	2 55 30·99	36 11 3·0	13·4	0·9293
2 12 9 31·6	R.	2 50 3·95	34 33 57·1	13·1	0·9257
4 11 50 3·7	W.	2 38 26·09	31 38 46·9	12·4	0·9183
11 10 36 20·2	W.	1 52 6·40	24 27 50·6	10·4	0·8946
13 10 14 20·7	R.	1 37 56·36	23 5 6·9	9·9	0·8891
14 10 3 16·7	W.	1 30 47·16	22 28 27·9	9·7	0·8865

In the computation of the parallaxes the adopted value of the Sun's mean horizontal parallax is 8″·85, and the geocentric distances, Δ, are taken from the *Astronomische Nachrichten*, No. 3,349.

Observers' Remarks.

April 30.—The comet appeared as a large diffused mass about 0′·8 in diameter, difficult to observe; there was no definite condensation. Only one bisection secured in N.P.D. Sky rather thick.

May 1.—Round patch of light; a condensation showed now and then, but difficult to bisect properly.

May 2.—Observed in a nearly dark field; observation not satisfactory; very difficult.

May 4.—Only a very approximate observation; sky very thick.

May 11.—Good observation.

May 13.—Very difficult, faint; sky hazy.

May 14.—Observation considered very fair for so faint an object.

Observers: W., Mr. W. Wickham; R., Mr. W. H. Robinson.

Radcliffe Observatory, Oxford:
1897 *March* 11.

Ephemerides for Physical Observations of Mercury.

By A. Marth.

Though the assumption of the coincidence of the plane of *Mercury's* equator with that of the orbit is arbitrary, it is at present the only one available, if the wishes of observers for ephemerides are to be complied with. I now give such ephemerides covering the times of Mr. Percival Lowell's observations made in the autumn of 1896, and also the part of the next synodical revolution, during which observations may be procurable. As it is simpler to employ in the computations the planet's geocentric ecliptical longitudes and latitudes, not published in the *Nautical Almanac*, I have used the places of the *Connaissance des Temps*, so that the data of the ephemeris are true ones for Paris noon, and apparent ones for Greenwich noon $+\tau$.

P denotes the position-angle of the assumed axis of rotation, reckoned, as usually, from the declination circle.

L + 180 and B, the longitude and latitude of the Earth referred to the planet's orbit, the longitude being reckoned from the perihelion.

ω_0 the hermographical longitude of the central meridian on the supposition that the period of rotation coincides with the planet's return to perihelion, or the angle "Mean Anomaly − L." If n is considered the daily rate of rotation "Const. $+ n(t - t_0)$ − L" must be substituted.

d, the apparent diameter, 6″·68 being the assumed value at distance 1.

q and Q, the amount and position-angle of the greatest defect of illumination, q being expressed in parts of the diameter.

v the angle between Sun and Earth, as seen from *Mercury*.

1896.		t m	P °	L °	B °	ω_0 °	d "	q	Q °	v °
Sept.	1	+0·1	28·53	108·10	+5·75	57·87	5·89	·270	115·5	62·59
	2	0·0	·61	109·45	5·78	60·62	5·96	·281	115·8	64·00
	3	−0·1	·68	110·77	5·80	63·39	6·03	·292	116·0	65·43
	4	0·2	·73	112·07	5·82	66·18	6·10	·304	116·2	66·88
	5	−0·4	28·77	113·34	+5·85	69·00	6·18	·316	116·4	68·36
	6	0·5	·80	114·58	5·87	71·85	6·26	·328	116·6	69·87
	7	0·6	·82	115·80	5·89	74·73	6·34	·341	116·8	71·41
	8	0·7	·83	116·98	5·92	77·64	6·43	·354	117·0	72·99
	9	−0·8	28·82	118·13	+5·94	80·59	6·52	·367	117·2	74·61
	10	1·0	·81	119·24	5·97	83·57	6·61	·381	117·4	76·29
	11	1·1	·79	120·32	5·99	86·58	6·71	·396	117·5	78·02
	12	1·2	·76	121·36	6·02	89·63	6·82	·412	117·7	79·81
	13	−1·3	28·73	122·35	+6·04	92·73	6·93	·428	117·8	81·67
	14	1·5	·69	123·31	6·07	95·87	7·04	·445	118·0	83·61
	15	1·7	·65	124·21	6·10	99·06	7·16	·462	118·2	85·62
	16	1·8	·61	125·06	6·12	102·30	7·28	·480	118·4	87·72
	17	−1·9	28·56	125·86	+6·15	105·60	7·41	·499	118·6	89·92
	18	2·0	·51	126·60	6·18	108·95	7·55	·519	118·8	92·22
	19	2·1	·46	127·27	6·20	112·37	7·69	·540	119·0	94·64
	20	2·3	·42	127·87	6·23	115·86	7·84	·563	119·2	97·19
	21	−2·4	28·38	128·39	+6·25	119·43	7·99	·586	119·5	99·87
	22	2·5	·34	128·84	6·28	123·08	8·15	·610	119·8	102·69
	23	2·7	·31	129·20	6·30	126·81	8·31	·635	120·1	105·67
	24	2·8	·29	129·46	6·32	130·64	8·48	·661	120·5	108·82
	25	2·9	·28	129·62	6·33	134·57	8·65	·689	120·9	112·16
	26	3·0	·29	129·67	6·34	138·61	8·82	·717	121·4	115·69
	27	−3·2	28·30	129·61	+6·35	142·77	9·00	·746	121·9	119·43
Oct.	15	−3·1	28·72	114·37	+5·75	231·67	8·91	·858	294·8	135·71
	16	2·9	·71	114·00	5·78	236·13	8·66	·815	295·8	129·15
	17	2·7	·70	113·83	5·80	240·39	8·40	·770	296·6	122·71
	18	2·5	·69	113·84	5·82	244·47	8·15	·722	297·2	116·41
	19	−2·3	28·69	114·04	+5·85	248·37	7·90	·673	297·6	110·27
	20	2·1	·69	114·41	5·87	252·09	7·65	·624	298·0	104·32
	21	1·9	·69	114·94	5·89	255·65	7·42	·575	298·3	98·58
	22	1·6	·68	115·62	5·92	259·06	7·20	·527	298·6	93·05
	23	−1·4	28·67	116·45	+5·94	262·33	6·99	·480	298·8	87·74

		t m	P	L	B	ω_0	d	q	Q	v
1896.			°	°	°	°	″		°	°
Oct.	24	1·2	·65	117·40	5·97	265·48	6·79	·436	298·9	82·66
	25	0 9	·62	118·45	5·99	268·52	6·61	·394	299·0	77·81
	26	0·7	·58	119·60	6·02	271·46	6·44	·355	299·1	73·19
	27	−0·5	28·52	120·83	+6·05	274·32	6·28	·319	299·1	68·81
	28	0·3	·45	122·13	6 07	277·11	6·13	·286	299·1	64·65
	29	−0·1	·36	123·49	6·10	279·84	6·00	·255	299·0	60·70
	30	+0·1	·25	124·90	6·12	282·52	5·88	·227	298·9	56·97
	31	0·3	28·12	126·36	6·15	285·16	5·76	·202	298·8	53·44
Nov.	1	0·5	27·98	127·85	6·18	287·76	5·66	·179	298·6	50·10
	2	+0·6	27·81	129·37	+6·20	290·33	5·56	·159	298·4	46·93
	3	0·8	27 62	130·91	6·23	292·88	5·47	·140	298·2	43·94
	4	1·0	27·41	132·47	6·25	295·41	5·38	·123	297·9	41·11
	5	1·1	27·17	134·05	6·28	297·93	5·31	·108	297·6	38·43
	6	+1·3	26·92	135·64	+6·30	300·43	5·24	·095	297·2	35·89
	7	1·4	26·64	137·24	6·32	302·93	5·17	·083	296·8	33·47
	8	1·5	26·35	138·84	6·33	305·42	5·11	·072	296·4	31·18
	9	1·6	26·03	140·45	6·34	307·90	5·06	·063	295·9	29·00
	10	+1·7	25·69	142·06	+6·35	310·38	5·01	·054	295·4	26·93
1897.										
Apr.	9	+1·2	332·39	312·23	−2·52	34·06	5·26	·054	67·4	26·78
	10	1·1	332 67	314·30	2·46	36·08	5·32	·071	66 8	30·94
	11	1·0	332·99	316·35	2·41	38·12	5·39	·091	66·4	35·17
	12	0·8	333·34	318·37	2·36	40·19	5·47	·114	66·2	39·46
	13	+0·7	333·71	320·37	−2·31	42·29	5·56	·139	66·1	43·81
	14	0·5	334·10	322·33	2·25	44·43	5·65	·167	66·1	48·17
	15	0·3	334·52	324·24	2·20	46·61	5·75	·196	66·2	52·54
	16	+0·1	334·95	326·10	2·14	48·83	5·86	·227	66·3	56·89
	17	−0·1	335·40	327·92	−2·09	51·11	5·98	·259	66·5	61·19
	18	0·3	335·86	329·68	2·03	53·45	6·11	·292	66·8	65·45
	19	0·5	336·32	331·37	1·97	55·85	6·24	·326	67·1	69·65
	20	0·7	336·79	333·00	1·91	58·31	6·38	·360	67·4	73·77
	21	−0·9	337·27	334·56	−1·84	60·84	6·53	·394	67·7	77·81
	22	1·1	337·74	336·05	1·77	63·44	6·69	·428	68·0	81·77
	23	1·3	338·20	337·46	1·69	66·12	6·86	·462	68·3	85·65
	24	1·5	338·65	338·80	1·61	68·88	7·04	·495	68·7	89·45
	25	−1·7	339·09	340·06	−1·53	71·71	7·22	·528	69·0	93·17
	26	1·9	339·51	341·24	1·44	74·62	7·41	·559	69·4	96·81
	27	2·1	339·92	342·34	1·35	77·61	7·61	·590	69·7	100·38
	28	2·2	340·31	343·36	1·25	80·69	7·82	·620	70·0	103·89

1897.		t m	P °	L °	B °	ω_0 °	d ″	q	Q °	v °
Apr.	29	−2·4	340·67	344·29	−1·14	83·85	8·03	·649	70·3	107·34
	30	2·6	341·01	345·14	1·02	87 09	8·25	·677	70·6	110·75
May	1	2·8	341·32	345·90	0·90	90·42	8·47	·704	70·9	114·12
	2	3·0	341·60	346·57	0·77	93·84	8·70	·730	71·2	117·44
	3	−3·1	341·85	347·16	−0·63	97·35	8·94	·755	71·5	120·73
	4	3·3	342·07	347·66	0·48	100·94	9·17	·779	71·8	124·01
	5	3·4	342·25	348·07	0·33	104 62	9·41	·803	72·0	127·28
	6	3·6	342·40	348·39	−0·17	108·39	9·65	·825	72·3	130·55
	7	3·7	342·52	348·62	+0·01	112 26	9·89	·846	72·5	133·81
	8	3·9	342·61	348·76	0·19	116·24	10·13	·866	72·8	137·08
	9	−4·0	342·65	348·82	+0·38	120·24	10·36	·885	73·1	140·37
	29	−4·5	339·86	340·21	+4·44	210·70	11·56	·944	241·3	152·52
	30	4·5	339·77	339·92	4·57	215·07	11·40	·931	242·0	149·49
	31	4·4	339·71	339·71	4·68	219·38	11·23	·917	242·6	146·51
June	1	4·3	339·67	339·57	4·79	223·62	11·05	·902	243·2	143·58
	2	−4·2	339·66	339·50	+4·88	227·78	10 85	·887	243·7	140·71
	3	4·1	339·67	339·51	4·96	231·86	10·65	·871	244·2	137 89
	4	4·0	339·70	339·59	5·03	235·87	10·45	·854	244·6	135 12
	5	3·9	339·76	339·75	5·09	239·80	10·24	·837	245·1	132·39
	6	−3·8	339·85	339·98	+5·15	243·66	10·03	·819	245·6	129·71
	7	3·7	339·96	340·29	5·19	247·44	9·82	·801	246·0	127·07
	8	3·6	340·10	340·68	5·23	251·15	9·61	·783	246·5	124·46
	9	3·4	340·27	341·14	5·26	254·78	9·40	·764	247 0	121·88
	10	−3·3	340·46	341·68	+5·28	258·34	9·19	·745	247·5	119·32
	11	3·2	340·68	342·29	5·29	261·82	8·98	·725	248·0	116·77
	12	3·0	340·93	342·97	5·30	265·23	8·77	·705	248·5	114·23
	13	2·9	341·21	343·72	5·30	268·58	8·57	·685	249·1	111·70
	14	−2·7	341·51	344·53	+5·30	271·86	8·38	·664	249·7	109·16
	15	2·6	341·85	345·41	5·30	275·07	8·19	·643	250·3	106·61
	16	2·4	342·21	346·36	5·29	278·21	8·00	·621	250·9	104·04
	17	2·2	342·61	347·37	5·28	281·29	7·81	·599	251·5	101·45
	18	−2·1	343·03	348·45	+5·26	284·31	7·63	·577	252·2	98·83
	19	1·9	343·49	349·58	5·24	287·27	7·46	·554	252·9	96·16
	20	1·7	343·98	350·78	5·22	290·16	7·29	·530	253·7	93·45
	21	1·6	344·51	352·04	5·19	292·99	7·13	·506	254·5	90·68
	22	−1·4	345·07	353·36	+5·17	295·76	6·97	·481	255·3	87·85
	23	1·2	345·66	354·74	5·14	298·48	6·82	·456	256·1	84·94
	24	1·0	346·29	356·17	5·11	301·14	6·67	·430	257·0	81·96
	25	0·8	346·96	357·66	5·09	303·74	6·53	·404	258·0	78·89

1897.		t m	P °	L °	B °	$\omega_\odot$ °	d °	q	Q °	v °
June	26	−0·7	347·67	359·21	+5·06	306·28	6·40	·377	259·0	75·73
	27	0·5	348·41	0·82	5·03	308·77	6·27	·350	260·0	72·48
	28	0·3	349·19	2·48	5·00	311·20	6·14	·322	261·1	69·12
	29	−0·2	350·01	4·19	4·97	313·58	6·03	·294	262·3	65·66
	30	0·0	350·86	5·96	4·95	315·90	5·92	·266	263·5	62·09
July	1	+0·2	351·75	7·78	+4·92	318·18	5·81	·238	264·8	58·42
	2	0·4	352·68	9·64	4·90	320·41	5·71	·211	266·2	54·65
	3	0·5	353·65	11·55	4·88	322·59	5·62	·184	267·7	50·78
	4	0·7	354·65	13·51	4·86	324·72	5·54	·158	269·2	46·82
	5	0·8	355·67	15·51	4·84	326·82	5·46	·133	270·9	42·79
	6	+1·0	356·72	17·54	+4·83	328·88	5·38	·110	272·7	38·69

I am quite willing to comply with the wishes of several observers and prepare similar ephemerides for *Venus* ; but as the sketches are scattered, and at present only partly accessible, I require information about the dates, and also references to the published sketches, so that they may be consulted when I get the opportunity.

Colonel Cooper's Observatory,
Markree, Collooney, Ireland.

Attempt to photograph the Leonids in the Open Sky.

By Isaac Roberts, D.Sc., F.R.S.

The first opportunity for attempting to photograph the *Leonid* meteor swarm occurred on the morning of Sunday last, February 28, between the hours of $0^h\ 30^m$ and $3^h\ 30^m$. The sky was fairly clear and the conditions generally favourable for the trial.

Two very sensitive plates were simultaneously exposed with their centres pointed to the part of the sky given in the ephemeris, and the exposure continued during $2^h\ 45^m$. The negatives are of good quality, and the one exposed in the 20-inch reflector shows stars to the faintness of about 17th magnitude, whilst the plate exposed in the camera with 5-inch portrait lens shows stars to 15th magnitude.

The photo-field of the reflector plate is 2°, and that of the camera plate 17°, in diameter.

On careful examination of the two plates I can detect no indication whatever of either the meteor stream or the comet that is expected to accompany it.

The conclusion at which I have arrived by these experiments is that the meteor stream, if it should at present be within a radius of $8\frac{1}{2}°$ with the computed position as a centre, does not emit or reflect light which would be equal to that of a star of the 15th magnitude, nor, if it is within a radius of 1° of the computed point as a centre, emit or reflect light that would be equal to that of a star of the 17th magnitude.

We may therefore conclude that the stream cannot be photographed during the present interval, but it is very desirable that another ephemeris should be computed in time for making a trial again next winter. The ephemeris should include the months of November, December, January, February, and March.

Errata in Mr. Tebbutt's Observations of Minor Planets.

Vol. lvi. page 352, line 14 from top *for* May 1 5·3 *read* May 18 5·3.

Vol. lvii. „ 26 „ 11 „ under Mean Time of *Nautical Almanac* *read* $0^h\ 0^m$.

Vol. lvii. page 26, line 17 from top under Mean Time of *Nautical Almanac* *for* $1^h\ 24^m$ *read* $1^h\ 24^m\ 0^s$.

Vol. lvii. page 30, line 7 from bottom *for* $10^h\ 3^m\ 20^s$ *read* $10^h\ 3^m\ 20{\cdot}5^s$.

„ „ 76 „ 9 „ top „ +1 1·21 „ +1 2·21.

„ „ 77 „ 5 „ „ „ 14 26 9·099 „ 14 26 9·09.

„ „ 77 „ 4 „ bottom „ +6 17·9 „ −6 17·9.

„ „ 79 „ 7 „ top „ +2 36·54 „ −2 36·54.

„ „ 79 „ 9 „ bottom „ 9·519*n* „ 9·510*n*.

„ „ 81 „ 5 „ top „ Radcliffe 1893 *read* Radcliffe 1890.

„ „ 81 „ 3 „ bottom „ 112 59 20·4 „ 112 59 20·0

Errata in Radcliffe Observations of Comet Perrine-Lamp, vol. lvii. page 181.

	Parallax in N.P.D.		Log. ($q \times \Delta$).
Feb.	23 (*a*) *for* 14″·1 *read* 15″·9.		*For* 0·7377 *read* 0·7897.
	(*b*) „ 14·2 „ 15·8.		„ 0·7416 „ 0·7862.
Mar.	3 (*d*) „ 6·0 „ 10·0.		„ 0·5077 „ 0·7267.
	16 (*g*) „ 4·6 „ 5·0.		„ 0·6509 „ 0·6955.
Apr.	9 (*j*) „ 2·5 „ 3·2.		„ 0·6617 „ 0·7634.
	10 (*k*)		„ 0·7140 „ 0·7189.

Errata in Professor Schur's Paper on the Diameter, &c., of Mars.

Vol. lvii. page 151, third number in last column, *for* 6·11 *read* 6·21.